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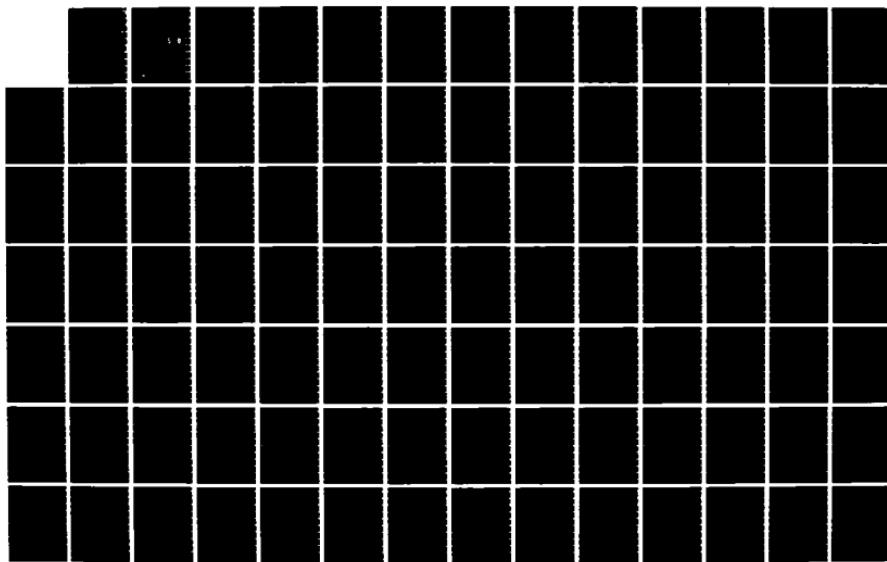
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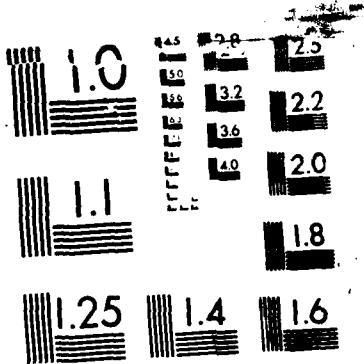
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USED IN THE
XYZ THREE-DIMENSIONAL POTENTIAL FLOW PROGRAM

An Engineering Report

by

WILLIAM JAMES BEARY JR.

1. N00228-85-G-3303



Submitted to the Faculty of
the College of Engineering
Texas A&M University

in partial fulfillment of the requirements for the degree of
MASTER OF ENGINEERING

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May 1986

Major Subject: Ocean Engineering

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ABSTRACT

The calculation of non-lifting potential flow about arbitrary three dimensional bodies is examined in detail with specific interest in the XYZ Potential Flow program developed by the David W. Taylor Naval Ship Research and Development Center. The program uses a surface singularity distribution to solve the Neumann boundary value problem by means of a source panel method assuming a flat element with a constant source density over the area of the element. Boundary conditions are applied at control points on the elements producing a system of linear equations for the source density. When the source density is known, velocities and pressure coefficients may be calculated.

The main purpose of this paper is to present the details of the approximation of an arbitrary three dimensional body using quadrilateral elements, and to provide a detailed derivation of the exact source panel integrations in order to gain insight for future research at Texas A&M University. A variation of the Hess method of surface discretization using quadrilateral source panels is described in detail as it is used in the XYZ Potential Flow program. The exact source panel integrations are derived in detail.

A general discussion of other aspects of the program is included. Velocities and pressure coefficients for flow about a triaxial ellipsoid are calculated using the XYZ Potential Flow Program solution, and the results are compared with the analytical solution and the Hess Program solution.



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1.0 INTRODUCTION

This paper examines two aspects of the development of the XYZ Potential Flow Program (hereafter referred to as the XYZPF Program), a FORTRAN program which uses a source panel method to approximate solutions to steady potential flow problems about arbitrary three dimensional bodies. The aspects examined in detail are (1) the description of the details of the approximation of an arbitrary three-dimensional body using quadrilateral elements, and (2) a detailed derivation of the exact source panel integrations.

The XYZPF Program was developed specifically for applications in numerical ship modelling and hydrodynamics studies at the David W. Taylor Naval Ship Research and Development Center (NSRDC) in Bethesda, Maryland. The format of the program is based on the work of Hess and Smith (1962) in the numerical calculation of non-lifting potential flow. A similar program is maintained by the Aerodynamics Division of the McDonnell-Douglas Corporation, referred to in this paper as the "Hess program." The XYZPF Program is a modification of what has come to be known generally as the Hess Method. The most significant modifications are improvements to the method of solving the matrix equation for the source density, and greater flexibility in the input options (Dawson and Dean 1972).

Though potential flow is a product of mathematics, and is never found in a real fluid, the results of potential flow calculations provide usable information for flow regions external to a thin boundary layer.

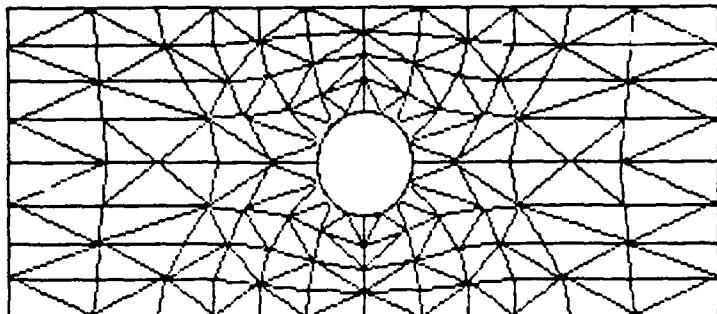
with little or no boundary layer separation. For such flow fields, the region outside the boundary layer may be considered to be effectively inviscid, and may be closely approximated by potential flow models. Small viscous effects can be accounted for by "thickening" the body by the appropriate displacement thickness. Displacement thickness accounts for the region of retarded fluid flow in the boundary layer inversely proportional to the square of the free stream velocity. Downstream of the point of boundary layer separation, the potential flow model no longer applies.

Prior to the development of numerical methods, analytical solutions were generally restricted to simple analytical shapes (Kellogg 1929). The need to solve boundary value problems for arbitrary boundaries in continuum mechanics has fostered the development of numerical approximations to the integral equation expressions. While the integration methods have been well known for quite some time, only since the advent of high speed computers have many of the problems been practical to solve by numerical methods. Among the numerical methods being used are finite differences, finite elements, and the boundary element method.

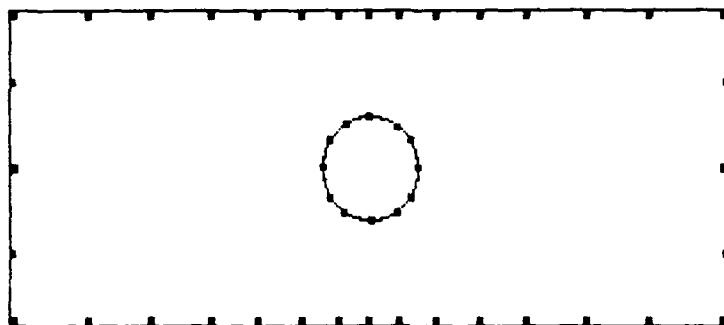
"Finite differences" and "finite elements" are numerical methods which satisfy the boundary conditions, and then approximate the solution to the governing equation in the fluid domain. These methods discretize the domain into a network of elements or cells.

Another approach is what is now known as the "Boundary Element Method," in which the governing equation is exactly satisfied in the domain, and the boundary conditions are applied through a boundary discretization method. The boundary value problem is reformulated as a boundary integral equation which is then discretized by subdividing the boundary into a finite number of surface elements. Each element is represented by an analytical function, and the source density function is integrated over the surface of each element. Two factors governing the accuracy of the boundary element method are the boundary discretization method and the source panel integration. These two factors are examined in detail in this report, as a detailed derivation of the exact source panel integration, including the development of the source panel geometry, has not previously appeared in literature.

The difference between the domain methods and the boundary methods is significant. The domain methods discretize the domain, while the boundary methods discretize the boundary. Thus, the boundary method reduces the dimension of the problem by one, as depicted in figure (1). In the application of the XYZPF Program, the problem is reduced from a three-dimensional problem in the domain to a two-dimensional problem on the boundaries. This method is well suited to problems in which the limits of the domain are infinite or difficult to define, in that the problem is applied to the boundary rather than the domain.



FINITE ELEMENT DISCRETIZATION



BOUNDARY ELEMENT DISCRETIZATION

Figure 1. Discretization Methods

Just as there are many variations of domain methods, there are also a variety of boundary methods. In general, they can be classified as "indirect" or "direct" formulations. The "indirect" method assumes a continuous source distribution over the surface of the body, and a solution which satisfies both the governing equation in the domain, and the boundary conditions on the body surface. The result is an integral equation on the boundary which has the surface source density function as its unknown. By enforcing the boundary conditions at control points on the surface, a system of equations is produced by which the source density may be determined.

The "direct" method solves the velocity potential function through

an application of Green's Second Identity requiring the solution of a source distribution and a dipole distribution on the boundary. The direct method has more physical significance to the general boundary value problem, and more versatility in its application as it can be applied to Neumann problems, Dirichlet problems, or mixed boundary value problems (Brebbia 1984).

The simplicity and accuracy of the indirect method has made it attractive for many applications. The source panel method is an application of the indirect formulation of the boundary element method to the Neumann type of potential flow problem, for which the normal derivative of the potential function is prescribed on the boundary.

1.1 OBJECTIVES

The purpose of this paper is (1) to describe the details of the approximation of an arbitrary three-dimensional body using quadrilateral elements, and (2) to provide a detailed derivation of the exact source panel integrations for use in future investigations at Texas A&M University using panels of higher order geometries and source density functions. This paper is not intended to be a user's manual, though a general discussion of other aspects of the program is also included. NSRDC Report 3892 (Dawson and Dean 1972) is a summary of the XYZPF Program for those strictly interested in its use.

2.0 HISTORICAL DEVELOPMENT

The foundations of the boundary element method were laid early in this century beginning with Fredholm in 1903 when he established the existence of solutions to the Neumann problem through a reconstruction of the problem using a discretized boundary (Kellogg 1929). The solution was determined to be the potential of a simple source distribution on a boundary with a continuous normal derivative for an infinite domain. Later works by Kellogg (1929) in potential theory demonstrated the application of the boundary integral equation method in electrostatics, heat transfer, flow in porous media, and fluid flow problems, but development was limited by the difficulty of obtaining analytical solutions. No significant advances were made until interest in boundary integral equation methods was revitalized with the advent of high speed electronic computers. Investigators were then able to discretize the boundaries and solve the integral equations numerically. This method of solution became known as the boundary element method. Early development of such numerical methods was pioneered by Hess and Smith (1962) and Jaswon and Symm (1963). Hess and Smith dealt primarily with the indirect formulation eventually leading to a solution for the three dimensional problem as described in this paper. In a parallel work, Jaswon and Symm developed a direct formulation approach to the two dimensional problem. The XYZPF Program is based primarily on the work of Hess and Smith. Hess has since developed a higher order panel method (Hess 1979) and Lefebvre modified the XYZPF Program for calculating velocity potentials for five degrees of freedom (Lefebvre 1982).

3.0 THEORETICAL DEVELOPMENT

3.1 THE POTENTIAL FLOW PROBLEM IN THREE DIMENSIONS

The governing equation for ideal (incompressible, inviscid, irrotational) flow is Laplace's equation:

$$\nabla^2 \Phi = 0 \quad (1)$$

where Φ is the velocity potential, and ∇^2 is the Laplacian operator. The XYZPF Program deals with steady, uniform flow of an ideal fluid about an arbitrary three dimensional body. The velocity components at any point within the flow field may be obtained from the negative gradient of the velocity potential, i. e.

$$\mathbf{V} = -\nabla \Phi = -\frac{\partial \Phi}{\partial x} \mathbf{i} - \frac{\partial \Phi}{\partial y} \mathbf{j} - \frac{\partial \Phi}{\partial z} \mathbf{k} \quad (2)$$

The freestream flow \mathbf{V}_∞ is defined as a uniform stream of unit magnitude.

$$|\mathbf{V}_\infty| = \sqrt{V_{\infty x}^2 + V_{\infty y}^2 + V_{\infty z}^2} = 1 \quad (3)$$

The key to the boundary element method is the Divergence Theorem (Green's Theorem) which relates a volume integral to an equivalent surface integral reducing the three-dimensional problem to a

two-dimensional one. The expression for Green's second identity is (Lamb 1924):

$$\iiint (\Phi \nabla^2 w - w \nabla^2 \Phi) d\Omega = \iint (w \frac{\partial \Phi}{\partial n} - \Phi \frac{\partial w}{\partial n}) d\Gamma \quad (4)$$

in which Ω represents the integration over the three dimensional domain, and Γ represents integration over the two dimensional boundary. The partial derivatives are taken with respect to the outward normal, n . The weighting function, w , is usually chosen to be the fundamental solution for three dimensions, $w = 1/(4\pi r)$, where r is the distance from the source to an arbitrary point on the boundary.

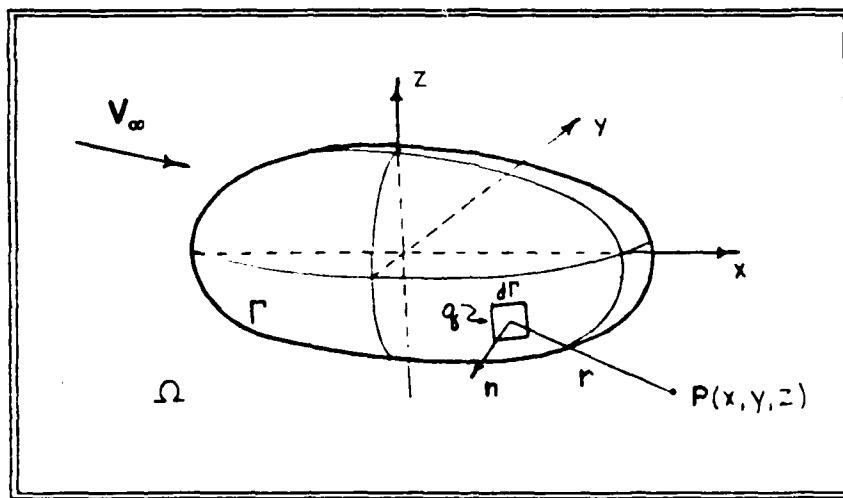


Figure 2. Potential Flow in Three Dimensions

Consider an arbitrary three-dimensional body with surface Γ , having an equation of the form $F(x, y, z) = 0$ where x, y, z are Cartesian coordinates of the global reference system as shown in Figure (2). The unit outward normal, n , at any point on the surface is given by the gradient of the function describing the surface divided by the magnitude

of the gradient, i.e.

$$\mathbf{n} = \frac{\pm \nabla \Phi}{|\nabla \Phi|} \quad (5)$$

where the sign of the unit normal vector is chosen to ensure that the vector is an outward normal. The potential function Φ describing the flow field must meet the following boundary conditions:

a. $\nabla^2 \Phi = 0$ (Laplace's Equation) (6)

b. For an impermeable boundary, the velocity normal to the surface must be zero relative to the boundary (the Neumann boundary condition):

$$\left(\frac{\partial \Phi}{\partial n} \right)_{\Gamma} = 0 \quad (7)$$

c. The velocity potential approaches the freestream velocity potential as the distance from the body goes to infinity:

$$\Phi \rightarrow \Phi_{\infty} \quad \text{as} \quad |\mathbf{r}| \rightarrow \infty \quad (8)$$

The total potential at any point in the domain is composed of the freestream potential and the disturbance potential due to the body,

$$\Phi = \Phi_{\infty} + \psi \quad (9)$$

The disturbance potential, ψ , satisfies the following boundary conditions:

a. $\nabla^2\psi = 0$ (10)

b. From equation (7), the velocity normal to the boundary due to the disturbance and due to the onset flow must be of equal magnitude, but opposite sign. Then from equation (9)

$$\left(\frac{\partial \psi}{\partial n} \right)_{\Gamma} = n(p) \cdot v_{\infty} \quad (11)$$

Note that the normal vector is a function of position on the surface of the body.

c. The disturbance potential approaches zero as the distance from the body goes to infinity, i. e.

$$\psi \rightarrow 0 \quad \text{as} \quad |\mathbf{r}| \rightarrow \infty \quad (12)$$

3.2 MATHEMATICAL MODEL

The disturbance potential of the body may be represented by a distribution of a source density function σ over the body surface. The potential at an arbitrary point $P(x, y, z)$ due to the surface potential is (Kellogg 1929):

$$\psi(x, y, z) = \iint \frac{\sigma(q)}{r(P,q)} d\Gamma \quad (13)$$

where q is the integration point on the surface, and

$$r(P,q) = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}$$

is the distance from the field point P to the integration point q .

The source density distribution function must satisfy the boundary conditions for the disturbance potential. Boundary conditions (10) and (12) are automatically satisfied by the form of the integrand. However, equation (11), the velocity normal to the boundary, combined with the Neumann boundary condition, equation (7), is the key to solving the boundary integral problem.

The integrand becomes singular as the surface of the body is approached, i. e. $|r|$ goes to zero. The singularity represents the local fluid flux normal to the boundary due to the local source density. The principal value of the singularity is $-2\pi\sigma(p)$, determined through a limiting process of the Gauss Flux Theorem (Kellogg 1929). The point p represents a field point which lies on the boundary. The integral expression is now composed of the contribution of the local source density and the contribution of the source density function over the remainder of the body surface. Solving for the velocity normal to the surface yields the following expression:

$$\left(\frac{\partial \psi}{\partial n} \right)_{\Gamma} = -2\pi\sigma(p) + \iint \frac{\partial}{\partial n} \left[\frac{\sigma(q)}{r(p,q)} \right] d\Gamma \quad (14)$$

From equation (11), this expression becomes:

$$2\pi\sigma(p) - \iint \frac{\partial}{\partial n} \left[\frac{\sigma(q)}{r(p,q)} \right] d\Gamma = -n(p) \cdot V_{\infty} \quad (15)$$

This equation is a two dimensional Fredholm integral equation of the second kind, which ensures a unique solution, and that the diagonal elements of the system matrix will be dominant, each having a value of 2π (Kellogg 1929). Once equation (15) has been solved for the source density σ , the velocity components at any point of the flow field may be obtained by differentiating the disturbance potential function (13) with respect to the coordinate directions and adding the components of the freestream flow, V_{∞} .

$$V(x, y, z) = V_{\infty} - \frac{\partial \psi}{\partial x} i - \frac{\partial \psi}{\partial y} j - \frac{\partial \psi}{\partial z} k \quad (16)$$

The body shape does not have to be slender, axisymmetric, or simply connected, allowing for analysis of interior flow and a wide range of applications of the method. The only restriction imposed on the form of the body is that it must have a continuous normal vector. Discontinuities in the right hand side of equation (15) will produce unwanted singularities. Thus, in the process of approximating a body which has distinct corners, where there is clearly a discontinuity in the normal vector, the corner must be replaced by a surface with some finite

curvature. However, application of this method has shown that the flow calculations give correct results for convex corners, while unrounded concave corners may or may not produce significant error, depending on the angle produced by the corner (Hess and Smith 1962).

Because of the method of approximation, the calculation of flow velocities on the body surface are restricted to the points at which the boundary conditions were applied. Velocities at points other than those must be obtained by interpolation. Direct calculation of velocities at the edge of an element yields infinite velocities.

With the solution of the system of linear equations for the source densities, the flow velocities at any point in the domain may be obtained from equation (16), and pressure coefficients are then computed from the velocities using a form of the Bernoulli equation:

$$P(t) = \frac{p}{\rho} + \frac{1}{2} |v|^2 + \frac{\partial \Phi}{\partial t} \quad (17)$$

where $P(t)$ is a constant independent of position. In the XYZPF Program, the flow is steady. Therefore, equation (17) can be reduced to

$$p + \frac{1}{2} \rho |v|^2 = \text{constant} \quad (18)$$

and the pressure field can be expressed in terms of a dimensionless pressure coefficient C_p as:

$$C_p = \frac{p - p_\infty}{\frac{1}{2} \rho |v_\infty|^2} = 1 - \frac{|v|^2}{|v_\infty|^2} \quad (19)$$

where p_∞ is the static pressure at infinity.

3.3 NUMERICAL MODEL

In order to represent the surface of a body in the domain mathematically, the body may be described by analytical expressions which may provide an exact representation of the surface. However, the types of bodies which can be adequately described by such methods are severely limited. Another way to represent the body is to use a large number of analytical expressions, each describing only a small portion of the body. Hess and Smith (1962) suggested the use of an assembly of flat quadrilateral elements to model the actual surface of the body, as shown in Figure (3). Each quadrilateral approximates a region of the surface described by points which lie on the actual surface of the body. As planar elements, these quadrilaterals are clearly analytical, and when carefully constructed, the elements can approximate arbitrary three dimensional body surfaces without restriction.

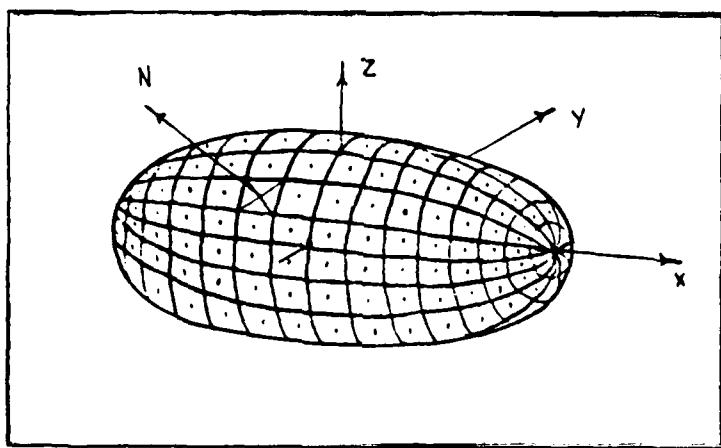


Figure 3. Approximation of the body by surface elements

The XYZPF Program uses the discretization procedure described by Hess and Smith (1962) with some minor modifications. The three dimensional body surface may be described using a large number of plane quadrilateral elements, each assumed to have a constant source density over the area of each element. Regions of the body requiring higher resolution for sharp curvature or anticipated velocity gradients will require a higher concentration of elements.

Because the plane quadrilateral elements cannot fit edge to edge on a rounded surface, small gaps in the panel approximation contribute to the error of the approximation. However, the error due to the gaps is negligible when compared with the error of the basic model, that is, using flat panels to approximate a curved surface. Triangular elements have been suggested in an attempt to eliminate the gaps (Levy 1959), but the increased accuracy is so small that it may not justify the additional work of organizing the triangular elements in lieu of the simpler quadrilaterals (Hess and Smith 1966). The method presented is valid for an polygonal element with any number of sides.

Equation (15) can now be decomposed into a summation of integrals, each representing the contribution of one element of the body surface. The unknown source density can be taken outside the integral, since it is assumed to be a constant over each element. The integration is performed over the area of the source element, and the boundary condition equation (11) is then enforced at a single point p in each remaining element. By performing this operation at each element of the

surface, a system of linear equations is generated which is equal in number to the number of surface elements and the number of unknown source densities. Equation (15) can now be approximated by the matrix equation (Dawson and Dean 1972):

$$\sigma_i = \sum_j \sigma_j C_{ij} + V_i \quad (20)$$

where

$$C_{ij} = \frac{1}{2\pi} \iint_j \frac{\partial}{\partial n_i} \left[\frac{1}{r_{ij}} \right] dA$$

$$C_{ii} = 0$$

$$V_i = -\frac{1}{2\pi} \left[n_i \cdot v_\infty \right]$$

It is important to note that the influence coefficients C_{ij} and C_{ii} are functions of geometry only, and once computed, need not be recomputed for analysis of several different flows. From the solution of equation (20) on the discretized surface, equation (13) may be applied at any point in the domain. Then, the velocity at an arbitrary field point $P(x, y, z)$ in the domain may be determined from equation (16). With the velocity known, the pressure coefficient is determined from equation (19).

4.0 ORGANIZATION OF THE PROGRAM

The XYZPF Program is actually composed of seven independent programs, referred to as sections PF1 through PF7, each of which builds on data generated from a previous section. This type of organization allows the user the flexibility of rerunning portions of the program using different flow parameters without having to go through the time consuming process of recalculating the influence coefficient matrix, which is dependent only on the geometry of the body. While the NSRDC program is very similar to the Hess program, there are also some significant differences. The following list of differences is taken from NSRDC Report 3892 (Dawson and Dean 1972):

- (1) The input to XYZ PF is arranged to facilitate the preparation of input for a series of problems in which only one part of the body is changed. Also, a number of checks are made on the input to help detect errors.
- (2) An option was added for the recomputation of the source density and velocities for only part of the body when only small changes are made. This option also provides for the use of the solution of one problem as an initial guess for the solution of another problem.
- (3) The matrix of influence coefficients is computed column by column instead of row by row. This column arrangement was used for the original LARC computer version because it required much less high speed memory. The computation is also about 10% faster this way than with the row-by-row arrangement.
- (4) A simultaneous displacement iterative scheme is used to solve

the matrix coefficient for the source density. The scheme is slower than the successive displacement (Gauss-Seidel) scheme used in the Hess program, but it can be carried out using the matrix column by column instead of row by row.

(5) When possible, an extrapolation procedure is used to reduce the number of iterations required for convergence. One such method is the Richardson extrapolation.

The methods used in the XYZPF Program will be discussed in detail in the following sections.

5.0 DETAILS OF THE SURFACE APPROXIMATION

5.1 PREPARATION OF THE INPUT

Section PF1 is set up to read and store the input data, and to examine the cornerpoints of the quadrilaterals to detect obvious errors in the input. Because of the number of points which may have to be entered for a complex geometry, the user input is a major source of program error, and this first look for input errors will save lot of run time in the program as a whole. If Section PF1 detects major errors in the input, the program will not continue with the calculation of the coefficient matrix, but will stop and identify the grid location of the error. Minor errors may not cause the program to stop, but will be noted in the output.

One of the major advantages to this program is in the organization of the input data. The surface is input in sections so that small portions of the input geometry may be changed without having to recalculated the points for the entire surface. The program also takes advantage of symmetry to minimize the input effort. Only the portion of the body which has no redundancy needs to be entered point by point. The remainder of the body is reflected across the planes of symmetry by the program to complete the surface representation.

The surface is represented by a set of points in three-dimensional space which lie on the actual surface, and which will later be used to define the plane quadrilateral source elements. These points are defined in the global reference system. The points on the surface should be

selected in such a way as to provide an accurate representation of the surface with the fewest number of points possible. Portions of the surface which are highly curved will require a larger number of points to provide adequate resolution. Additionally, portions of the surface in which the flow field is expected to change rapidly will require a large number of points to accurately determine the flow field in that region. Some familiarity with fluid dynamics will provide a somewhat intuitive approach to properly distributing the elements. Elements should change gradually in size from areas of high concentration to those of just a few large elements, changing no more than 50 percent in size between adjacent elements (Hess and Smith 1966). The accuracy is only as good as that provided by the largest element in a particular area. The use of quadrilateral elements facilitates the use of known analytical equations and body contours to determine the input points.

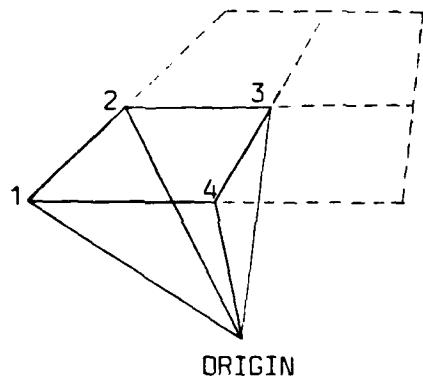


Figure 4. The 3D quadrilateral element in global coordinates

For the purposes of this program, the body surface is represented by a large number of plane quadrilateral elements as shown in figure (3), each of which is assumed to have a constant value of source density over the area of the element. Each element is defined by four input points

which lie on the actual surface as shown in figure (4). Since each input point can be used as a corner for up to four elements, there is no need to enter the same point four separate times. The input points are organized in groups of four to form the quadrilateral element, and each point may also be associated with adjacent quadrilaterals. This is accomplished through the use of a two dimensional coordinate system in which the user assigns a pair of integers, m and n, to each point which identifies the "row" and the "column" in which it lies. A column of points will be given a common value of n, and each point in that column will have a unique value of m corresponding to the row in which the point lies. The orientation of these "coordinate" integers determines the direction of the outward normal for each element. Looking from the flow field toward the section of elements, if the values of m are increasing upward, the values of n must increase to the right. Increasing m and n can point in any direction with respect to the global reference system. In fact, the orientation can change from one section to another. However, any other relationship between m and n will produce an incorrect normal vector. Once assigned, the values of m and n also serve to identify the element for which the corresponding point is a corner. The four points which form a quadrilateral element are two points of one column, or n line, with consecutive m numbers, and two points of the next higher n line with the same m numbers as the previous two points. Thus, the element m, n is composed of the points identified by (m, n), (m+1, n), (m, n+1), and (m+1, n+1).

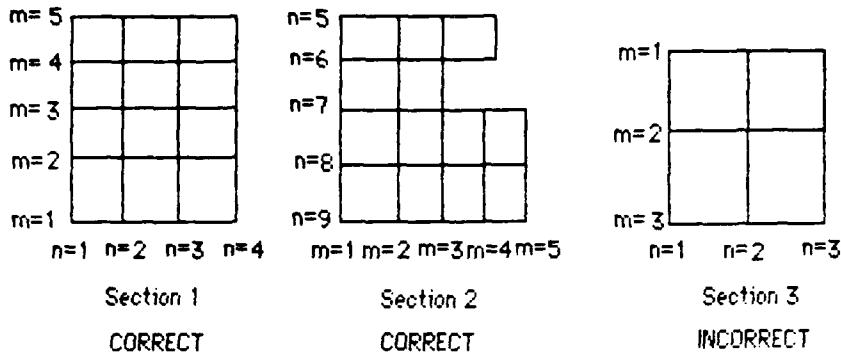


Figure 5. Quadrilateral index numbers

Each section of the body surface is formed by specifying a set of corner points corresponding to the m, n pairs for all of the quadrilaterals of the section. The user will sequentially assign an m number to the points for each n line, and also number the n lines for the section points entered. The first point in each n line will always have $m = 1$. The n lines are also numbered sequentially, but the value of n is not reset for each new section. The sequence of n numbers runs through all the sections as shown in figure (5). Points on a particular row or column do not have to be strictly colinear. By forming nearly triangular elements, a rounded planform can be approximated without conflicting with the numbering convention, as shown in figure (6).

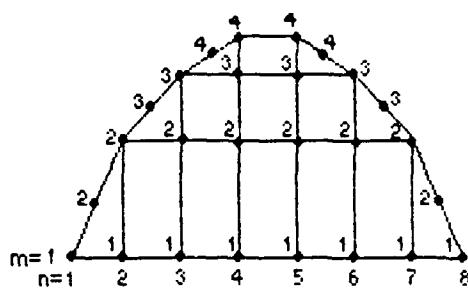


Figure 6. Approximating a thin region with rounded planform (Hess & Smith 1962)

By entering data in sections, small changes in geometry can be performed without having to reenter all the points associated with the body surface. This feature is unique to the XYZPF Program, and offers a great deal of flexibility in design work. However, with the added flexibility comes more restrictions on both the input of the original geometry and on any modifications. There are four important restrictions on the input which are required to provide quadrilateral elements in groups of four to facilitate geometry calculations (Dawson and Dean 1972):

(1) There must be an even number of elements in both the m and n directions in each section of the body.

(2) The common corner point of a group of four elements must not coincide with any other corner point. The sides between the elements serve to define the local coordinate system, and serve as the axis of rotation when the surface is flattened for numerical differentiation of

the velocity potential.

(3) Each set of four elements must have at least seven distinct corner points to allow the elements to more closely conform to a curved surface. This also allows for convergence of N-lines or M-lines as may occur, for example, at the leading edge of an ellipsoid. Thus, only two of the four quadrilateral may degenerate into triangles by having two of their corner points coincide. This does not necessarily eliminate the possibility of more than two "triangular" elements since the adjacent sides of a quadrilateral may be colinear as shown in figure (6).

(4) The normal vectors between two adjacent quadrilaterals in a group of four must be less than 90 degrees and preferably less than 45 degrees. If a sharp edge is required, it should be a concave corner with respect to the flow field, and the input should be arranged so that the edge is along an outside boundary of the groups of four, and not through the center.

When making small changes to the original geometry, the number of elements used in a new section must be the same as the number used in the original section unless the part being changed is at the end of the input data. Section configurations may be selected by natural divisions, as a matter of convenience to more easily handle large numbers of points, or as a tool to take advantage of symmetry.

In setting up input data to use planes of symmetry, it is important to note that the XYZPF Program has certain restrictions on the choice of

symmetry planes. The user only has the option to select the number of symmetry planes. The planes which will be used as symmetry planes are preselected by the program to optimize the calculation procedure. Therefore, knowing this, the preparation of input data must consider the following restrictions imposed by the program (Dean and Dawson 1972).

If only one plane of symmetry is used, the plane of symmetry is the $y = 0$ plane of the global coordinate system. As such, all the y coordinates of the input points must be of the same sign, i. e., all positive or all negative. If the body is closed and intersects the plane of symmetry, the points touching the plane, i. e., corresponding to $y = 0$, must also be entered with the input points.

If two planes of symmetry are used, the planes of symmetry are the $y = 0$ plane and the $z = 0$ plane in the global coordinate system. Again, the y coordinates of all input points must have the same sign, positive or negative, and the z coordinates of all points must be of the same sign, positive or negative without regard to the sign of y . If the body surface intersects one or both of the planes of symmetry, the points which lie in the plane, i. e., those corresponding to $y = 0$ or $z = 0$, must also be entered with the input points.

If three planes of symmetry are used, clearly the planes are the reference planes of the global coordinate system. As with the previous cases, all the x coordinates of the input points must be of the same sign, and similarly for the y and z coordinates. If any part of the body intersects any of the planes of symmetry, the points which lie in that

plane, i. e., $x = 0$, $y = 0$ or $z = 0$, must also be entered with the input points.

5.2 SOURCE PANEL GEOMETRY

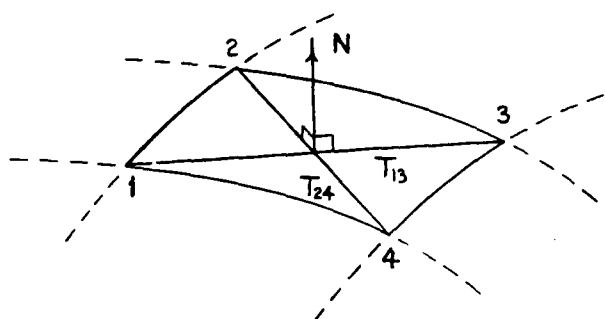


Figure 7. The outer normal to the quadrilateral element

With the surface points identified by the location numbers, m and n, and arranged in accordance with program requirements, calculation of various aspects of the source panel geometry and formation of the plane quadrilateral element is the next step in the numerical integration process. Formation of all of the planar elements is identical, so the following discussion of source panel geometry will deal with only one characteristic element. The four corner points forming the basic quadrilateral are numbered in a clockwise direction from 1 to 4 as shown in figure (7). It does not matter which corner point is identified with the number 1 subscript, but the remaining points must be numbered consecutively in a clockwise direction when observed from the flow field in order to ensure an outward directed normal vector. These subscripts will be used to identify the corner points for the remainder of this discussion. For this example, the points will be identified as follows:

<u>Position Numbers</u>	<u>Global Coordinates</u>
m, n	X_1, Y_1, Z_1
m+1, n	X_2, Y_2, Z_2
m+1, n+1	X_3, Y_3, Z_3
m, n+1	X_4, Y_4, Z_4

In forming the plane quadrilateral elements, the corner points, which are generally not coplanar, are used to form the local coordinate system, relative to the element. Recalling that the cross product of two vectors yields a vector solution which is perpendicular to both of the original vectors, the normal to the element may be obtained from the cross product of the diagonals of the element,

$$\mathbf{N} = \mathbf{T}_{24} \times \mathbf{T}_{13} \quad (21)$$

where \mathbf{T}_{13} is the vector from point 1 to point 3, and \mathbf{T}_{24} is the vector from corner point 2 to point 4. The unit normal is then:

$$\mathbf{n} = \frac{\mathbf{T}_{24} \times \mathbf{T}_{13}}{|\mathbf{T}_{24} \times \mathbf{T}_{13}|} \quad (22)$$

This unit normal now represents the first of the three local coordinate directions, this one in the ζ direction. The side of the quadrilateral from point 2 to point 3 is then used to obtain the second coordinate vector.

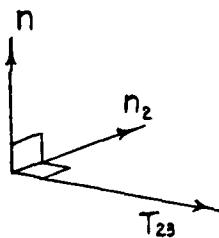


Figure 8. The second local coordinate vector

$$\mathbf{N}_2 = \mathbf{n} \times \mathbf{T}_{23} \quad (23)$$

and the unit vector

$$\mathbf{n}_2 = \frac{\mathbf{N}_2}{|\mathbf{N}_2|} \quad (24)$$

Similarly, the third local coordinate vector is obtained from the crossproduct of \mathbf{n}_2 and \mathbf{n} , the result of which is a unit vector.

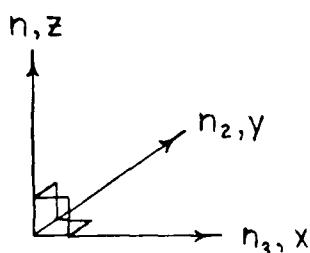


Figure 9. The third local coordinate vector

$$\mathbf{n}_3 = \mathbf{n}_2 \times \mathbf{n} \quad (25)$$

The unit vectors \mathbf{n}_3 , \mathbf{n}_2 , and \mathbf{n} form an orthonormal basis and define the local coordinate system for the element in the ξ , η , and ζ directions

respectively. Other methods of obtaining an orthonormal basis could be used just as well, and would make no difference to the remaining computations. The origin of the local coordinate system would most correctly be located at the "null point," the point at which the velocity potential has no contribution to the tangential velocity component on the source element. The null point is the point in each quadrilateral element where the normal velocity boundary condition is applied. However, with the exception of long, thin quadrilaterals, the physical difference between the null point and the centroid of the quadrilateral is not significant. The XYZPF Program will print a warning in the output when a quadrilateral is long and thin enough to jeopardize the accuracy of the approximation in that region. By locating the origin of the local coordinate system at the centroid, rather than at the null point, the difficult process of locating the null point for each element can be eliminated, later calculations of the multipole expansion can be simplified, and the boundary conditions can be applied at the centroid without contributing significant error to the approximation (Hess and Smith 1966). Therefore, the origin for each local coordinate system is located at the centroid for the respective element.

5.3 LOCATING THE CENTROID

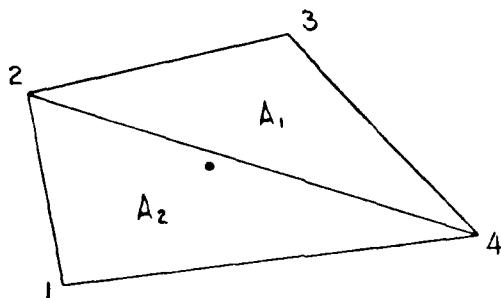


Figure 10. Locating the centroid of the quadrilateral

The centroid of the element may be calculated by first dividing the area of the quadrilateral into two triangular areas, the triangles being separated by the line from point 2 to point 4. The area A_1 of the triangle defined by corner points 2, 3, and 4 is

$$A_1 = \frac{1}{2} |T_{24} \times T_{23}| \quad (26)$$

Similarly, the area A_2 of the triangle defined by corner points 1, 2, and 4 is

$$A_2 = \frac{1}{2} |T_{12} \times T_{14}| \quad (27)$$

In the global coordinate system, the X component of the centroid is given by

$$\bar{x} = \frac{A_1 \bar{x}_1 + A_2 \bar{x}_2}{A_1 + A_2} \quad (28)$$

where \bar{x}_1 and \bar{x}_2 are the averages of the X components of the corner points of each triangle. Substituting the values for \bar{x}_1 and \bar{x}_2 :

$$\begin{aligned}
 \bar{x} &= \frac{\frac{1}{3}A_1(x_2+x_3+x_4) + \frac{1}{3}A_2(x_1+x_2+x_4)}{A_1 + A_2} \\
 &= \frac{1}{3} \left[\frac{(A_1+A_2)x_2 + (A_1+A_2)x_4 + A_1x_3 + A_2x_1}{A_1 + A_2} \right] \\
 &= \frac{1}{3} \left[x_2 + x_4 + \frac{A_1x_3 + A_2x_1}{A_1 + A_2} \right]
 \end{aligned} \tag{29}$$

Similarly

$$\bar{y} = \frac{1}{3} \left[y_2 + y_4 + \frac{A_1y_3 + A_2y_1}{A_1 + A_2} \right] \tag{30}$$

$$\bar{z} = \frac{1}{3} \left[z_2 + z_4 + \frac{A_1z_3 + A_2z_1}{A_1 + A_2} \right] \tag{31}$$

5.4 COORDINATE TRANSFORMATION

Now that the local coordinate system is formed and properly located at the centroid of the element, the global coordinates of the corner points (X, Y, Z) are transformed to local coordinates (ξ, η, ζ) through the components of the reference vectors of the local coordinate system as follows:

$$\begin{bmatrix} n_{3x} & n_{3y} & n_{3z} \\ n_{2x} & n_{2y} & n_{2z} \\ n_x & n_y & n_z \end{bmatrix} \begin{bmatrix} x - \bar{x} \\ y - \bar{y} \\ z - \bar{z} \end{bmatrix} = \begin{bmatrix} \xi \\ \eta \\ \zeta \end{bmatrix} \tag{32}$$

The corner points are projected into the plane of the quadrilateral element by setting the ζ components to zero. The original diagonal vectors, T_{13} and T_{24} , will be on opposite sides of the resulting plane. The plane quadrilateral element is now completely defined. The program will sweep through all of the input elements using the assigned location numbers, and repeat this process for each element.

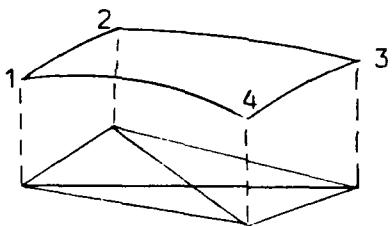


Figure 11. Forming the plane quadrilateral element

5.5 MOMENTS OF INERTIA

The calculation of the moments of inertia for each element are performed for use in the computation of the velocity coefficients using the quadrupole method. Any calculus text will give the moments of inertia of a planar section with a constant unit density about the origin to be:

$$I_{xx} = \iint_A \xi^2 d\xi d\eta \quad (33)$$

$$I_{yy} = \iint_A \eta^2 d\xi d\eta \quad (34)$$

$$I_{xy} = \iint_A \xi \eta d\xi d\eta \quad (35)$$

For the triangular region defined by the corner points 2, 3, and 4,

$$I_{xx} = \frac{A}{12} [(\xi_2 + \xi_3)^2 + (\xi_3 + \xi_4)^2 + (\xi_4 + \xi_2)^2] \quad (36)$$

$$I_{yy} = \frac{A}{12} [(\eta_2 + \eta_3)^2 + (\eta_3 + \eta_4)^2 + (\eta_4 + \eta_2)^2] \quad (37)$$

$$I_{xy} = \frac{A}{12} [(\xi_2 + \xi_3)(\eta_2 + \eta_3) + (\xi_3 + \xi_4)(\eta_3 + \eta_4) + (\xi_4 + \xi_2)(\eta_4 + \eta_2)] \quad (38)$$

Similar equations can be generated for the triangular region defined by the corner points 1, 2, and 4. The moment of inertia for the entire quadrilateral is the sum of the corresponding expressions for each of the triangles. The resulting equations are:

$$I_{xx} = \frac{A}{12} [(\xi_2 + \xi_3)^2 + (\xi_3 + \xi_4)^2 + (\xi_4 + \xi_2)^2] + \frac{A}{12} [(\xi_1 + \xi_2)^2 + (\xi_2 + \xi_4)^2 + (\xi_4 + \xi_1)^2] \quad (39)$$

$$I_{yy} = \frac{A}{12} [(\eta_2 + \eta_3)^2 + (\eta_3 + \eta_4)^2 + (\eta_4 + \eta_2)^2] + \frac{A}{12} [(\eta_1 + \eta_2)^2 + (\eta_2 + \eta_4)^2 + (\eta_4 + \eta_1)^2] \quad (40)$$

$$I_{xy} = \frac{A}{12} [(\xi_2 + \xi_3)(\eta_2 + \eta_3) + (\xi_3 + \xi_4)(\eta_3 + \eta_4) + (\xi_4 + \xi_2)(\eta_4 + \eta_2)] + \frac{A}{12} [(\xi_1 + \xi_2)(\eta_1 + \eta_2) + (\xi_2 + \xi_4)(\eta_2 + \eta_4) + (\xi_4 + \xi_1)(\eta_4 + \eta_1)] \quad (41)$$

6.0 THE MATRIX OF INFLUENCE COEFFICIENTS

With the quadrilaterals completely formed, the next step is to calculate the velocities induced by the elements at the centroids of all the other elements. The total number of elements forming the surface will be represented by N. Let the source element be the (j)th element, and the element for which the velocity components are to be calculated at the centroid is the (i)th element. It does not matter how the (i)th elements are arranged in relation to each other as the sequence progresses. However, the sequence must be consistent as the calculations proceed from one source element to another. This program sweeps through the (i)th elements in the order of their location numbers, m and n. For each consecutive n line, the elements are swept in order of increasing m numbers.

The result of the induced velocity calculations for the elements with unit source densities is an N by N square matrix of the values of induced velocities at each element, known also as the "matrix of influence coefficients." The XYZ potential flow program calculates the coefficients column by column, while the Hess program calculates them row by row. The advantage of one over the other depends on the method of later solving the matrix for the source densities. In calculating the influence coefficients, twenty-five quantities which describe the geometry of the source element are required to adequately calculate the induced velocity at the centroid of the (i)th element. These quantities include the coordinates of the centroid in the global coordinate system, the elements of the coordinate transformation matrix, the local

coordinates of the corner points, the maximum diagonal, the area, and the second moments of the quadrilateral element. Additionally, the Hess program uses the coordinates of the null point, making a total of twenty-eight quantities for that method (Hess and Smith 1962).

When calculating row by row, the first (i)th element is selected, containing the "null" point, and the influence coefficients are computed for all of the (j)th elements in sequence before proceeding to the (i+1)th element. This procedure requires the twenty-five quantities for each (j)th element to be available for calculation of the influence coefficient. Because each of the N (j)th elements is used N times with this procedure, calculating the geometric quantities or retrieving the values from low speed memory would be very time consuming, since the calculations or memory access would need to be performed N^2 times. Therefore, it is more practical to have the values available in high speed memory, although large matrices may exceed the storage capacity of high speed memory, imposing a limit on the number of elements which can be used. The advantage to the row-by-row calculation is that solution of the resulting matrix by the Gauss-Seidel reduction method does not require transposing the matrix, which would be another time consuming process (Hess and Smith 1962).

Another alternative is calculation of the influence coefficients column by column. This method calculates the influence coefficients by sweeping all the (i)th elements for each (j)th element before proceeding to the (j+1)th element. Therefore, the twenty-five geometric quantities are retrieved from low speed storage only once for each (j)th element.

for a total of N times. This procedure is not limited by the capacity of high speed memory, and calculation of the coefficient matrix is approximately 10% faster than the row-by-row method (Dawson and Dean 1972). This is the calculation method used by the XYZ Potential Flow Program.

An influence coefficient represents the combined effects on one element of the velocity potentials of all the other elements comprising the body surface. For the quadrilateral element with a unit source density in the xy -plane, from equation (13), the potential at point $P(x, y, z)$ due to the element is

$$\psi = \iint_A \frac{1}{r} dA = \iint_A \frac{d\xi d\eta}{\sqrt{(x - \xi)^2 + (y - \eta)^2 + z^2}} \quad (42)$$

The integrand, $1/r$, can be expanded in a series about the origin in powers of ξ and η . Each term of the series will contain the product of some powers of ξ and η with a corresponding derivative of $1/r_0$, where r_0 is the distance of the field point P from the quadrilateral origin.

$$r_0 = \sqrt{x^2 + y^2 + z^2}$$

and let

$$w = \frac{1}{r_0}$$

Then the series expansion through the second order term is (Hess and Smith 1962):

$$\Psi = Aw - (M_x w_x + M_y w_y) + 1/2(I_{xx} w_{xx} + 2I_{xy} w_{xy} + I_{yy} w_{yy}) + \dots \quad (43)$$

The subscripts, x and y , used with w represent the respective partial derivatives. This series represents the multipole expansion of the velocity potential, since each term can be interpreted as a point singularity of a particular order. The first term is the potential at point P due to a point source of strength A located at the origin. The second term is the sum of two dipoles of strengths M_x and M_y located at the origin, oriented along the x and y axis respectively. The choice of the centroid of the quadrilateral as the origin of the local coordinate system causes the first moments, M_x and M_y , to be zero. Therefore, the dipole terms disappear, and are not dealt with anywhere in the program. The third term is the sum of three quadrupoles of strengths I_{xx} , I_{xy} , and I_{yy} located at the origin. Kellogg (1929) shows that this second order approximation is absolutely and uniformly convergent, and Hess and Smith (1962) show that convergence is rapid enough with an increase in r_0 that certain further approximations may be made without significant error at large distances r_0 from the source quadrilateral.

Hess and Smith (1962) presented a comparison of velocities calculated using the exact formulas, a simple point source, and a second order approximation. The comparisons were based on the ratio of the distance r_0 , between the centroid of the source quadrilateral and the field point P , to the length of the maximum dimension t , of the source quadrilateral, typically the maximum diagonal. The non-dimensional ratio is then r_0/t . The results show that sufficient accuracy is maintained

while using a simple point source at ratios of $(r_0/t) \geq 4$, using the second order source and quadrupole solution for the range $2.45 \leq (r_0/t) < 4$, and using the exact solution for ratios of $(r_0/t) < 2.45$. In any case, the error goes to infinity as the field point approaches the edge of the quadrilateral where calculations indicate an infinite velocity. The XYZ Potential Flow Program uses a monopole source for $(r_0/t) > 4$, the source - quadrupole formulae for $2 < (r_0/t) \leq 4$, and the exact formulae for $(r_0/t) \leq 2$. Hess and Smith (1962) reported a maximum error of 0.001 in approximating any velocity component using the above criteria.

7.0 DERIVATION OF THE EXACT SOURCE PANEL INTEGRATION

From equations (2) and (42), the components of the velocity at the field point $P(x, y, z)$ due to the source quadrilateral are:

$$v_x = - \frac{\partial \psi}{\partial x} = \iint_A \frac{(x - \xi) d\xi d\eta}{r^3} \quad (44)$$

$$v_y = - \frac{\partial \psi}{\partial y} = \iint_A \frac{(y - \eta) d\xi d\eta}{r^3} \quad (45)$$

$$v_z = - \frac{\partial \psi}{\partial z} = \iint_A \frac{z d\xi d\eta}{r^3} \quad (46)$$

Equations (44), (45) and (46) are evaluated by expressing each of the integrals as the sum of four terms, each term representing the effect of one side of the quadrilateral (Hess and Smith 1962). This method can also be generalized for polygonal elements with any number of sides. The potential function for each side of the quadrilateral is the combined potentials of semi-infinite strips whose boundaries are the side of the quadrilateral and two semi-infinite lines parallel to either the x or y axis. When observed from the domain, and the sides are traversed in a clockwise direction, the source strip on the right will have a source density of $\sigma = +1/2$ and the source strip on the left will have a source density of $\sigma = -1/2$ as shown in figure (12). When the sides are recombined to form the quadrilateral, the source densities outside the quadrilateral cancel each other, and the source densities within the quadrilateral combine to form a source density of $\sigma = +1$. This will be

true for a planar element with any number of sides and in any relative orientation within the plane.

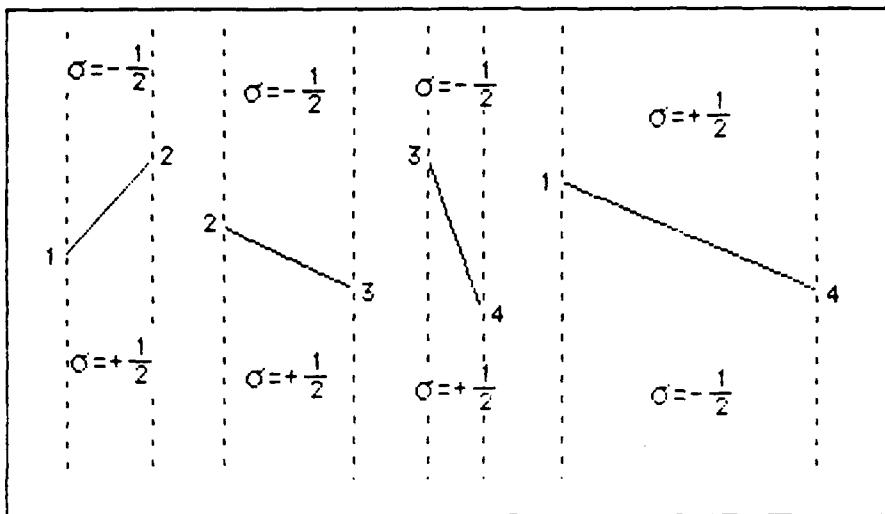


Figure 12. Fundamental potential functions for sides of a quadrilateral (Hess & Smith 1962)

7.1 THE Y VELOCITY COMPONENT

From equation (45), the velocity component V_y is found by summing the four terms representing the contributions of the sides of the quadrilateral. For the side from point (ξ_1, η_1) to point (ξ_2, η_2) , the contribution is expressed as the integral over the area of the semi-infinite strips with the source densities of $\sigma = +1/2$ and $\sigma = -1/2$ rather than the unit source density of equation (45).

$$V_{y12} = \int_{\xi_1}^{\xi_2} d\xi \left[\frac{1}{2} \int_{-\infty}^{\eta_{12}} - \frac{1}{2} \int_{\eta_{12}}^{\infty} \right] \frac{(y - \eta) d\eta}{r^3} \quad (47)$$

$$V_{y12} = \frac{1}{2} \int_{\xi_1}^{\xi_2} d\xi \left[\int_{-\infty}^{\eta_{12}} - \int_{\eta_{12}}^{\infty} \right] \frac{(y - \eta) d\eta}{[(x - \xi)^2 + (y - \eta)^2 + z^2]^{3/2}}$$

Integrating with respect to η :

$$V_{y12} = \frac{1}{2} \int_{\xi_1}^{\xi_2} d\xi \left\{ \frac{1}{[(x - \xi)^2 + (y - \eta)^2 + z^2]^{1/2}} \Big|_{\eta_{12}} \right. \\ \left. - \frac{1}{[(x - \xi)^2 + (y - \eta_1)^2 + z^2]^{1/2}} \Big|_{-\infty} - \frac{1}{[(x - \xi)^2 + (y - \eta_1)^2 + z^2]^{1/2}} \Big|_{\infty} \right. \\ \left. + \frac{1}{[(x - \xi)^2 + (y - \eta)^2 + z^2]^{1/2}} \Big|_{\eta_{12}} \right\}$$

The terms evaluated at $\eta = +\infty$ and $\eta = -\infty$ cancel, and the terms evaluated at $\eta = \eta_{12}$ add to obtain the following expression:

$$V_{y12} = \frac{1}{2} \int_{\xi_1}^{\xi_2} \frac{2 d\xi}{[(x - \xi)^2 + (y - \eta)^2 + z^2]^{1/2}}$$

$$V_{y12} = \int_{\xi_1}^{\xi_2} \frac{d\xi}{r} \quad (48)$$

Equation (48) is changed to a function of arclength s by the relation

$$\frac{d\xi}{ds} = \frac{\xi_2 - \xi_1}{\sqrt{(\xi_2 - \xi_1)^2 + (\eta_2 - \eta_1)^2}} = \frac{\xi_2 - \xi_1}{d_{12}} \quad (49)$$

where d_{12} is the length of the side of the quadrilateral from (ξ_1, η_1) to (ξ_2, η_2) as shown in figure (13).

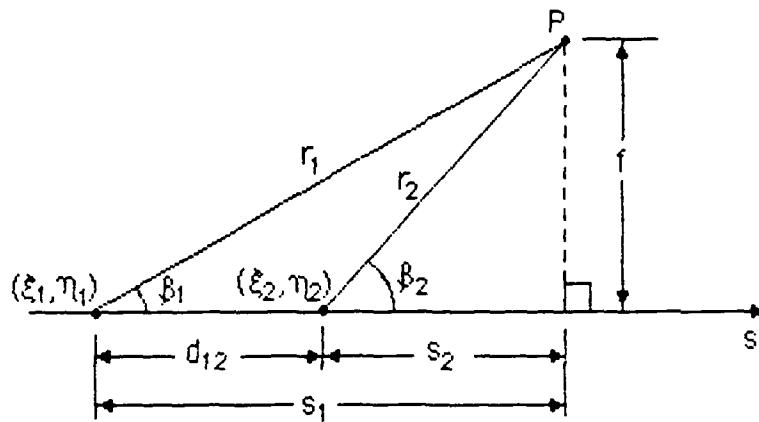


Figure 13. The potential due to a finite line source (Hess & Smith 1962)

Substituting equation (49) into equation (48)

$$V_{y_{12}} = \frac{\xi_2 - \xi_1}{d_{12}} \int_0^{d_{12}} \frac{ds}{r} \quad (50)$$

From figure (13), it can be seen that, in terms of arclength s , the distance r from point P to any point on the line from point 1 to point 2 is given by

$$r = \sqrt{f^2 + (s_1 - s)^2}$$

Substituting into equation (50) yields

$$\begin{aligned} V_{y_{12}} &= \frac{\xi_2 - \xi_1}{d_{12}} \int_0^{d_{12}} \frac{ds}{\sqrt{f^2 + (s_1 - s)^2}} \\ &= \frac{\xi_2 - \xi_1}{d_{12}} \int_0^{d_{12}} \frac{ds}{\sqrt{f^2 + (s - s_1)^2}} \end{aligned} \quad (51)$$

Evaluating the integral

$$\begin{aligned}
 V_{y_{12}} &= \frac{\xi_2 - \xi_1}{d_{12}} \log \left[(s - s_1) + \sqrt{f^2 + (s - s_1)^2} \right] \Big|_0^{d_{12}} \\
 &= \frac{\xi_2 - \xi_1}{d_{12}} \left\{ \log \left[(d_{12} - s_1) + \sqrt{f^2 + (d_{12} - s_1)^2} \right] \right. \\
 &\quad \left. - \log \left[(-s_1) + \sqrt{f^2 + (-s_1)^2} \right] \right\} \\
 &= \frac{\xi_2 - \xi_1}{d_{12}} \left\{ \log(r_2 - s_2) - \log(r_1 - s_1) \right\} \\
 &= \frac{\xi_2 - \xi_1}{d_{12}} \log \frac{(r_2 - s_2)}{(r_1 - s_1)} \tag{52}
 \end{aligned}$$

The quantities r_1 , r_2 , s_1 , and s_2 used in equation (52) are as shown in figure (13). Equation (52) is singular when $r_1 = s_1$, which occurs when the field point P is located anywhere along the line defined by the side of the quadrilateral. This singularity may be removed by using the law of cosines (Hess and Smith 1962).

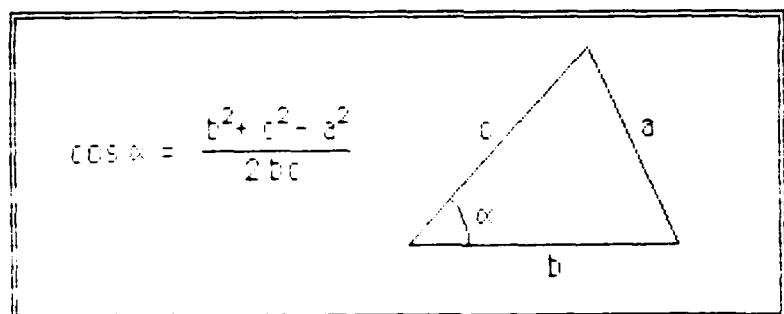


Figure 14. The law of cosines

From equation (52)

$$V_{y_{12}} = \frac{\xi_2 - \xi_1}{d_{12}} \log \frac{r_2(1 - \cos \beta_2)}{r_1(1 - \cos \beta_1)} \quad (53)$$

where β_1 and β_2 are the interior angles shown in figure (13). Applying the law of cosines to figure (13)

$$\cos \beta_1 = \frac{r_1^2 + d_{12}^2 - r_2^2}{2r_1 d_{12}} \quad (54)$$

$$\cos \beta_2 = -\frac{r_2^2 + d_{12}^2 - r_1^2}{2r_2 d_{12}} = \frac{r_1^2 - d_{12}^2 - r_2^2}{2r_2 d_{12}} \quad (55)$$

From equations (54) and (55):

$$\begin{aligned} \frac{r_2(1 - \cos \beta_2)}{r_1(1 - \cos \beta_1)} &= \frac{r_2 \left[1 - \frac{r_1^2 - d_{12}^2 - r_2^2}{2r_2 d_{12}} \right]}{r_1 \left[1 - \frac{r_1^2 + d_{12}^2 - r_2^2}{2r_1 d_{12}} \right]} \\ &= \frac{r_2 \left[\frac{2r_2 d_{12} - r_1^2 + d_{12}^2 + r_2^2}{2r_2 d_{12}} \right]}{r_1 \left[\frac{2r_1 d_{12} - r_1^2 - d_{12}^2 + r_2^2}{2r_1 d_{12}} \right]} \\ &= \frac{2r_2 d_{12} - r_1^2 + d_{12}^2 + r_2^2}{2r_1 d_{12} - r_1^2 - d_{12}^2 + r_2^2} = \frac{(r_2 + d_{12})^2 - r_1^2}{-(r_1 - d_{12})^2 + r_2^2} \\ &= \frac{[(r_2 + d_{12}) + r_1][(r_2 + d_{12}) - r_1]}{[r_2 + (r_1 - d_{12})][r_2 - (r_1 - d_{12})]} = \frac{r_1 + r_2 + d_{12}}{r_1 + r_2 - d_{12}} \end{aligned} \quad (55)$$

Substituting equation (56) into equation (53) yields the final form of the exact equation of the y component of the velocity induced by the side of the quadrilateral from point 1 to point 2:

$$V_{y_{12}} = \frac{\xi_2 - \xi_1}{d_{12}} \log \frac{r_1 + r_2 + d_{12}}{r_1 + r_2 - d_{12}} \quad (57)$$

Equation (57) is applied to the remaining sides of the quadrilateral simply by substituting the appropriate point numbers for the corner points of each side. The total contribution of the quadrilateral to the y component of the velocity is the sum of the four terms representing the contributions of each of the sides. The y component of the velocity at the field point P is now given by:

$$V_y = \frac{\xi_2 - \xi_1}{d_{12}} \log \frac{r_1 + r_2 + d_{12}}{r_1 + r_2 - d_{12}} + \frac{\xi_3 - \xi_2}{d_{23}} \log \frac{r_2 + r_3 + d_{23}}{r_2 + r_3 - d_{23}} \\ + \frac{\xi_4 - \xi_3}{d_{34}} \log \frac{r_3 + r_4 + d_{34}}{r_3 + r_4 - d_{34}} + \frac{\xi_1 - \xi_4}{d_{41}} \log \frac{r_4 + r_1 + d_{41}}{r_4 + r_1 - d_{41}} \quad (58)$$

7.2 THE X VELOCITY COMPONENT

A similar derivation process is used to produce the equation for the x component of the velocity induced by the side of the quadrilateral from point 1 to point 2. The semi-infinite source strips are constructed parallel to the x axis, and the order of integration is reversed.

The x component of the velocity at the field point P due to the quadrilateral is given by:

$$v_x = \frac{\eta_2 - \eta_1}{d_{12}} \log \frac{r_1 + r_2 + d_{12}}{r_1 + r_2 - d_{12}} + \frac{\eta_3 - \eta_2}{d_{23}} \log \frac{r_2 + r_3 + d_{23}}{r_2 + r_3 - d_{23}} \\ + \frac{\eta_4 - \eta_3}{d_{34}} \log \frac{r_3 + r_4 + d_{34}}{r_3 + r_4 - d_{34}} + \frac{\eta_1 - \eta_4}{d_{41}} \log \frac{r_4 + r_1 + d_{41}}{r_4 + r_1 - d_{41}} \quad (59)$$

7.3 THE Z VELOCITY COMPONENT

The z component of the velocity at the field point P due to the quadrilateral is obtained in a similar fashion, using semi-infinite source strips, this time parallel to the y axis. From equation (46), the fundamental velocity potential of the semi infinite source strips is integrated in a manner similar to that used to obtain equation (47), and the z component of the velocity due to the side from (ξ_1, η_1) to (ξ_2, η_2) is given by

$$v_{z12} = \int_{\xi_1}^{\xi_2} d\xi \left[\frac{1}{2} \int_{-\infty}^{\eta_{12}} - \frac{1}{2} \int_{\eta_{12}}^{\infty} \right] \frac{z d\eta}{r^3} \quad (60)$$

$$v_{z12} = \frac{z}{2} \int_{\xi_1}^{\xi_2} d\xi \left[\int_{-\infty}^{\eta_{12}} - \int_{\eta_{12}}^{\infty} \right] \frac{d\eta}{[(x - \xi)^2 + (y - \eta)^2 + z^2]^{\frac{3}{2}}}$$

Performing the integration with respect to η , the integral

$$\int \frac{d\eta}{[(x - \xi)^2 + (y - \eta)^2 + z^2]^{\frac{3}{2}}}$$

fits the integral form

$$-\int \frac{dF}{[C^2 + F^2]^n} = \frac{-1}{2C^2(n-1)} \left[\frac{F}{[C^2 + F^2]^{n-1}} + (2n-3) \int \frac{dF}{[C^2 + F^2]^{n-1}} \right]$$

where

$$C^2 = (x - \xi)^2 + z^2$$

$$F = (y - \eta)$$

$$dF = -d\eta$$

$$n = 3/2$$

Then, from equation (60)

$$\begin{aligned} V_{z_{12}} &= -\frac{z}{2} \int_{\xi_1}^{\xi_2} d\xi \left\{ \frac{(y - \eta)}{[(x - \xi)^2 + z^2][(x - \xi)^2 + (y - \eta)^2 + z^2]^{1/2}} \Big|_{\eta_{12}} \right. \\ &\quad - \frac{(y - \eta)}{[(x - \xi)^2 + z^2][(x - \xi)^2 + (y - \eta)^2 + z^2]^{1/2}} \Big|_{-\infty} \\ &\quad - \frac{(y - \eta)}{[(x - \xi)^2 + z^2][(x - \xi)^2 + (y - \eta)^2 + z^2]^{1/2}} \Big|_{\infty} \quad (61) \\ &\quad \left. - \frac{(y - \eta)}{[(x - \xi)^2 + z^2][(x - \xi)^2 + (y - \eta_{12})^2 + z^2]^{1/2}} \Big|_{\eta_{12}} \right\} \end{aligned}$$

Again, the terms evaluated at $+\infty$ and $-\infty$ cancel and the terms evaluated at η_{12} add to obtain the following expression:

$$V_{z_{12}} = -z \int_{\xi_1}^{\xi_2} \frac{(y - \eta_{12}) d\xi}{[(x - \xi)^2 + z^2][(x - \xi)^2 + (y - \eta_{12})^2 + z^2]^{1/2}} \quad (62)$$

Without a convenient substitution with which to integrate equation (62), the integration is performed directly. Recognizing that along the line defined by the side of the quadrilateral from (ξ_1, η_1) to (ξ_2, η_2) , η_{12} may be expressed as a function of ξ :

$$\eta_{12} = m_{12} \xi + b_{12} \quad (63)$$

where the slope of the side, m_{12} is given by

$$m_{12} = \frac{\eta_2 - \eta_1}{\xi_2 - \xi_1} \quad (64)$$

and b_{12} may be determined knowing that $\eta_{12} = \eta_1$ when $\xi = \xi_1$.

$$b_{12} = \frac{\xi_2 \eta_1 - \xi_1 \eta_2}{\xi_2 - \xi_1} \quad (65)$$

Substituting equation (63) into equation (62) yields

$$V_{z12} = -z \int_{\xi_1}^{\xi_2} \frac{(y - m_{12}\xi - b_{12}) d\xi}{\xi_1 [(x - \xi)^2 + z^2] [(x - \xi)^2 + (y - m_{12}\xi - b_{12})^2 + z^2]^{\frac{1}{2}}} \quad (66)$$

Define the quantities

$$q_{12} = y - b_{12} - m_{12}x \quad (67)$$

$$u = x - \xi \quad (68)$$

Then $du = -d\xi$ (69)

$$y - b_{12} - m_{12}\xi = m_{12}u + q_{12} \quad (70)$$

By a change of variable, equation (66) is now expressed as a function of u :

$$V_{z,12} = z \int_{x-\xi_1}^{x-\xi_2} \frac{(m_{12}u + q_{12}) du}{[u^2 + z^2][u^2 + (m_{12}u + q_{12})^2 + z^2]^{\frac{1}{2}}} \quad (71)$$

$$= z \int_{x-\xi_1}^{x-\xi_2} \frac{(m_{12}u + q_{12}) du}{[u^2 + z^2][(m_{12}^2 + 1)u^2 + 2m_{12}q_{12}u + q_{12}^2 + z^2]^{\frac{1}{2}}} \quad (72)$$

which fits the form of

$$\int \frac{(Lu + M) du}{(Au^2 + 2Bu + C)\sqrt{(au^2 + bu + c)}} \quad (73)$$

where

$$L = m_{12}$$

$$M = q_{12}$$

$$A = 1$$

$$a = (m_{12}^2 + 1)$$

$$B = 0$$

$$b = m_{12}q_{12}$$

$$C = z^2$$

$$c = q_{12}^2 + z^2$$

From Hardy (1944), this integral form may be integrated by the substitution

$$u = \frac{\mu t + v}{t + 1} \quad (74)$$

where μ and v satisfy

$$a\mu v + b(\mu + v) + c = 0 \quad (75)$$

$$A\mu v + B(\mu + v) + C = 0 \quad (76)$$

- and are the roots of the equation

$$(aB - bA)\xi^2 - (cA - aC)\xi + (bC - cB) = 0 \quad (77)$$

Substituting the appropriate values into equation (75), the roots of the quadratic equation are

$$\mu = -\frac{q_{12}}{m_{12}} \quad (78)$$

$$v = \frac{m_{12}z^2}{q_{12}} \quad (79)$$

It can be verified that these values satisfy equations (75) and (76).

Substituting equations (78) and (79) into equation (74)

$$u = \frac{\frac{m_{12}z^2}{q_{12}} - \frac{q_{12}t}{m_{12}}}{t + 1} \quad (80)$$

$$du = \left[\frac{-\frac{q_{12}t}{m_{12}} - \frac{m_{12}z^2}{q_{12}}}{(t + 1)^2} \right] dt \quad (81)$$

By substitution and a change of variable, equation (72) becomes a function of the parameter t . After simplification, the integral now fits the form of

$$K \int \frac{dt}{(\alpha t^2 + \beta) \sqrt{(\gamma t^2 + \delta)}} \quad (82)$$

where

$$K = -(m_{12}^6 q_{12} z^4 + 2m_{12}^4 q_{12}^3 z^2 + q_{12}^5 m_{12}^2)$$

$$\alpha = q_{12}^4 + m_{12}^2 q_{12}^2 z^2$$

$$\beta = m_{12}^4 z^4 + m_{12}^2 q_{12}^2 z^2$$

$$\gamma = q_{12}^4 + m_{12}^2 q_{12}^2 z^2$$

$$\delta = m_{12}^6 z^4 + m_{12}^4 z^4 + 2m_{12}^4 q_{12}^2 z^2 + m_{12}^2 q_{12}^4 + m_{12}^2 q_{12}^2 z^2$$

Equation (82) can be rationalized by the substitution

$$v = \frac{t}{\sqrt{\gamma t^2 + \delta}} \quad (83)$$

from which it can be shown that

$$t^2 = \frac{v^2 \delta}{1 - v^2 \gamma} \quad (84)$$

$$dt = \left[\frac{\delta}{(1 - v^2 \gamma)^{3/2}} \right]^{1/2} dv \quad (85)$$

Substituting equations (84) and (85) into equation (82) and simplifying yields the integral in terms of the parameter v :

$$K \int \frac{dt}{(\alpha t^2 + \beta) \sqrt{\gamma t^2 + \delta}} = K \int \frac{dv}{\beta + (\alpha \delta - \beta \gamma) v^2} \quad (86)$$

which fits the form

$$\int \frac{dv}{a^2 + b^2 v^2} = \frac{1}{ab} \tan^{-1} \frac{bv}{a} \quad (87)$$

where $a^2 = \beta$

$$b^2 = (\alpha \delta - \beta \gamma)$$

Performing the integration

$$K \int \frac{dy}{\beta + (\alpha\delta - \beta\gamma)y^2} = \frac{K}{\sqrt{\beta(\alpha\delta - \beta\gamma)}} \tan^{-1} \left[y \sqrt{\frac{\alpha\delta - \beta\gamma}{\beta}} \right] \quad (86)$$

From equations (80), (83), and the expressions for α , β , δ , and γ from equation (82), and after a considerable amount of algebraic manipulation and simplification, equation (88) becomes

$$\begin{aligned} & \frac{K}{\sqrt{\beta(\alpha\delta - \beta\gamma)}} \tan^{-1} \left[y \sqrt{\frac{\alpha\delta - \beta\gamma}{\beta}} \right] \\ &= -\frac{1}{z} \tan^{-1} \left[\frac{m_{12}z^2 - q_{12}u}{z \sqrt{z^2 + u^2 + (m_{12}u + q_{12})^2}} \right] \end{aligned} \quad (89)$$

From equations (63) and (67), equation (89) becomes

$$\begin{aligned} & -\frac{1}{z} \tan^{-1} \left[\frac{m_{12}z^2 - q_{12}u}{z \sqrt{z^2 + u^2 + (m_{12}u + q_{12})^2}} \right] \\ &= -\frac{1}{z} \tan^{-1} \left[\frac{m_{12}(u^2 + z^2) - (y - \tau_{l12})u}{z \sqrt{u^2 + (y - \tau_{l12})^2 + z^2}} \right] \end{aligned} \quad (90)$$

Finally, applying these results to equation (71)

$$\begin{aligned}
 V_{z12} &= z \int_{x-\xi_1}^{x-\xi_2} \frac{(m_{12}u + q_{12}) du}{[u^2 + z^2][u^2 + (m_{12}u + q_{12})^2 + z^2]^{\frac{1}{2}}} \\
 &= -\tan^{-1} \left[\frac{m_{12}(u^2 + z^2) - (y - \eta_{12})u}{z \sqrt{u^2 + (y - \eta_{12})^2 + z^2}} \right] \Big|_{x-\xi_1}^{x-\xi_2} \\
 &= \tan^{-1} \left[\frac{m_{12}((x - \xi_1)^2 + z^2) - (y - \eta_{12})(x - \xi_1)}{z \sqrt{(x - \xi_1)^2 + (y - \eta_{12})^2 + z^2}} \right] \\
 &\quad - \tan^{-1} \left[\frac{m_{12}((x - \xi_2)^2 + z^2) - (y - \eta_{12})(x - \xi_2)}{z \sqrt{(x - \xi_2)^2 + (y - \eta_{12})^2 + z^2}} \right] \tag{91}
 \end{aligned}$$

Recall that when $x = \xi_1$, $y = \eta_1$ and when $x = \xi_2$, $y = \eta_2$. Then, for the sake of a more compact equation, define the following quantities:

$$\begin{aligned}
 e_1 &= (x - \xi_1)^2 + z^2 & e_2 &= (x - \xi_2)^2 + z^2 \\
 h_1 &= (y - \eta_1)(x - \xi_1) & h_2 &= (y - \eta_2)(x - \xi_2)
 \end{aligned}$$

The quantities r_1 and r_2 are as shown in figure (12), where

$$r_1 = \sqrt{(x - \xi_1)^2 + (y - \eta_1)^2 + z^2} \quad r_2 = \sqrt{(x - \xi_2)^2 + (y - \eta_2)^2 + z^2}$$

Substituting these quantities into equation (91) yields the exact z component of velocity due to the side from point (ξ_1, η_1) to (ξ_2, η_2) in the form used by the XYZ Potential Flow Program:

$$V_{z12} = \tan^{-1} \left[\frac{m_{12}e_1 - h_1}{z r_1} \right] - \tan^{-1} \left[\frac{m_{12}e_2 - h_2}{z r_2} \right] \tag{92}$$

The total z component of the velocity at the field point P(x, y, z) due to the quadrilateral element is the sum of the four sides:

$$\begin{aligned}
 v_z = & \tan^{-1} \left[\frac{m_{12}e_1 - h_1}{z r_1} \right] - \tan^{-1} \left[\frac{m_{12}e_2 - h_2}{z r_2} \right] \\
 & + \tan^{-1} \left[\frac{m_{23}e_2 - h_2}{z r_2} \right] - \tan^{-1} \left[\frac{m_{23}e_3 - h_3}{z r_3} \right] \\
 & + \tan^{-1} \left[\frac{m_{34}e_3 - h_3}{z r_3} \right] - \tan^{-1} \left[\frac{m_{34}e_4 - h_4}{z r_4} \right] \\
 & + \tan^{-1} \left[\frac{m_{41}e_4 - h_4}{z r_4} \right] - \tan^{-1} \left[\frac{m_{41}e_1 - h_1}{z r_1} \right]
 \end{aligned} \tag{93}$$

8.0 APPROXIMATIONS OF THE INDUCED VELOCITY

8.1 QUADRUPOLE METHOD

As previously mentioned, as the ratio of (r_0/t) exceeds the value of 2, then certain approximations may be made which greatly reduce the calculation effort otherwise required by the exact method. In the range of $2 < (r_0/t) \leq 4$, the XYZ Potential Flow Program uses the second order approximation of the potential described by equation (43). With the origin at the centroid of the quadrilateral, the first moments are zero, and the second order approximation is

$$\phi = Aw + (1/2)(I_{xx}w_{xx} + 2I_{xy}w_{xy} + I_{yy}w_{yy}) \quad (94)$$

where the first term is a point source of strength A, the second term is composed of three quadrupoles of strengths I_{xx} , I_{xy} , and I_{yy} located at the local origin, and the subscripts on w indicate the partial derivatives of w with respect to those variables as before. The quantity A is the area of the element, and the terms I_{xx} , I_{xy} , and I_{yy} are the respective moments of inertia of the source element given by equations (39), (40), and (41). To obtain the velocity components at the field point, equation (94) is differentiated with respect to the coordinate directions giving:

$$V_x = - \frac{\partial \psi}{\partial x} = - \left[Aw_x + \frac{1}{2} I_{xx} w_{xxx} + I_{xy} w_{xxy} + \frac{1}{2} I_{yy} w_{xyy} \right] \quad (95)$$

$$V_y = - \frac{\partial \psi}{\partial y} = - \left[Aw_y + \frac{1}{2} I_{xx} w_{xx_y} + I_{xy} w_{xyy} + \frac{1}{2} I_{yy} w_{yyy} \right] \quad (96)$$

$$V_z = - \frac{\partial \psi}{\partial z} = - \left[Aw_z + \frac{1}{2} I_{xx} w_{xxz} + I_{xy} w_{xyz} + \frac{1}{2} I_{yy} w_{yyz} \right] \quad (97)$$

Recalling that $w = \frac{1}{\sqrt{x^2 + y^2 + z^2}} = \frac{1}{r_0}$

the derivatives of w , as expressed by Hess and Smith (1962) and as used in the XYZPF program, are

$$\left. \begin{aligned} w_x &= -x r_0^{-3} \\ w_y &= -y r_0^{-3} \\ w_z &= -z r_0^{-3} \end{aligned} \right\} \quad (99)$$

$$\left. \begin{aligned} w_{xxx} &= 3x(3p + 10x^2) r_0^{-7} \\ w_{xxy} &= 3y p r_0^{-7} \\ w_{xyy} &= 3x q r_0^{-7} \\ w_{yyy} &= 3y(3q + 10y^2) r_0^{-7} \\ w_{xxz} &= 3z p r_0^{-7} \\ w_{xyz} &= -15xyz r_0^{-7} \\ w_{yyz} &= 3z q r_0^{-7} \end{aligned} \right\} \quad (100)$$

where

$$p = y^2 + z^2 - 4x^2$$

$$q = x^2 + z^2 - 4y^2$$

8.2 MONPOLE METHOD

When the ratio of (r_0/t) is greater than 4, then the quadrilateral may be approximated by a simple source corresponding to the first term of equation (43). Then the velocity components at the field point due to the quadrilateral are given by

$$V_x = - \frac{\partial \psi}{\partial x} = - Aw_x \quad (101)$$

$$V_y = - \frac{\partial \psi}{\partial y} = - Aw_y \quad (102)$$

$$V_z = - \frac{\partial \psi}{\partial z} = - Aw_z \quad (103)$$

where the partial derivatives of w are those given in equation (99).

9.0 SOLVING THE MATRIX EQUATION FOR SOURCE DENSITY

9.1 JACOBI'S ITERATIVE METHOD

From equation (20), the matrix equation may be solved for the constant source density σ_i for each element which satisfies the boundary condition equation (11). Equation (20) suggests the use of Jacobi's iterative method of matrix solution in the form

$$\sigma_i^{(m+1)} = V_i + \sum_{\substack{j=1 \\ j \neq i}}^N C_{ij} \sigma_j^{(m)}, \quad i = 1, 2, \dots, N \quad (104)$$

where N is the number of elements composing the body surface, and m is the number of iterations completed. A partial sum of equation (104) is computed for each of the i th elements before proceeding to the next j th element. The iteration is complete when the summation of equation (104) includes all of the j th elements. Because the values of the source densities at all of the elements are recomputed before any of them are used in the iteration, this method is also called the simultaneous displacement method (Ralston 1965). This is contrasted with the Gauss-Seidel iterative method used in the Douglas program. In the Gauss-Seidel method, as each new σ_i is computed, it is used immediately in the iteration process for calculation of $\sigma_{(i+1)}$. This is also known as the successive displacement method and is expressed as

$$\sigma_i^{(m+1)} = v_i + \sum_{j=1}^{i-1} C_{ij} \sigma_j^{(m+1)} + \sum_{j=i+1}^N C_{ij} \sigma_j^{(m)} \quad (105)$$

$i = 1, 2, \dots, N$

Though the Gauss-Seidel iterative method is faster, the Jacobi iteration method was selected for use in the XYZPF program in order to be able to perform the iterations column by column, since the coefficient matrix is also computed column by column, and the matrix does not have to be transposed for solution.

When the $(m+1)$ th iteration is complete, the values of the source densities are compared with those of the (m) th iteration and the differences summed for all of the elements. The total difference between successive iterations is then compared to a convergence criteria input by the user. If the difference is less than the convergence criteria, then the matrix solution is complete and the values of the source densities are stored for later use in computing velocities and pressure coefficients. If the convergence criteria is not met, then the iteration process is repeated. After every five iterations, if the convergence criteria is still not met, then an extrapolation is attempted in order to accelerate the convergence. The XYZ Potential Flow Program uses a Richardson extrapolation method, a numerical procedure which uses two approximate results to obtain a third approximation which is closer to the exact solution (Ralston 1965).

9.2 RICHARDSON EXTRAPOLATION

The Richardson extrapolation assumes that the iterative process is convergent. For the iterative solutions S_0 , S_1 , and S_2 , where S_0 is the most recent approximation and S_2 the oldest, the solution is convergent if

$$\frac{S_0 - S_1}{S_1 - S_2} = \lambda < 1 \quad (106)$$

While a Richardson-type extrapolation can take many forms, the XYZPF program uses a procedure developed from the following approximations (Dawson and Dean 1972). If there is only one dominant eigenvalue and a sufficient number of iterations have been completed, the iterative solutions may be approximated by

$$S_0 \approx S_f + E \lambda^m$$

$$S_1 \approx S_f + E \lambda^{m-1} \quad (107)$$

$$S_2 \approx S_f + E \lambda^{m-2}$$

$$S_i \approx S_f + E \lambda^{m-i}$$

where S_f is the true solution

λ is the eigenvalue

E is the eigenfunction

m is the number of completed iterations

Define the linear combination which, from equation (107), may be

approximated as

$$AS_0 + (1 - A)S_1 \approx S_f + E\lambda^{n-1} (A\lambda + 1 - A) \quad (108)$$

The value of A may be chosen such that

$$(A\lambda + 1 - A) = 0 \quad (109)$$

Then, from equations (108) and (109)

$$AS_0 + (1 - A)S_1 \approx S_f \quad (110)$$

where the expression on the left converges to the exact solution.

From equations (106) and (109)

$$\lambda = \frac{S_0 - S_1}{S_1 - S_2} = 1 - \frac{1}{A} \quad (111)$$

Solving for A,

$$A = \frac{S_2 - S_1}{S_0 - 2S_1 + S_2} = \frac{S_2 - S_1}{D} \quad (112)$$

Since the value of A generally changes from element to element, a weighted average of A is used in the extrapolation, where

$$\bar{A} = \frac{\sum_{i=1}^N (S_2(i) - S_1(i)) (\text{sign of } D(i))}{\sum_{i=1}^N D(i)} \quad (113)$$

Equation (113) is recomputed after every fifth iteration. If the difference between the new value and the old value is less than 0.02, then the solution is extrapolated. From equation (110), the extrapolated solution is

$$S^* = \bar{A}S_0 + (1 - \bar{A})S_1 \quad (114)$$

When there are two dominant eigenvalues, then the iterative solutions may be approximated by

$$S_i \approx S_f + E_1 \lambda_1^{m-i} + E_2 \lambda_2^{m-i} \quad (115)$$

where S_f is the true solution

λ_1 and λ_2 are the eigenvalues

E_1 and E_2 are the eigenfunctions

m is the number of completed iterations

Define the linear combination which, from equation (115), may be approximated as

$$\begin{aligned} & B_2 S_0 + B_1 S_1 + (1 - B_1 - B_2) S_2 \\ & \approx S_f + E_1 \lambda_1^{m-2} [B_2 \lambda_1^2 + B_1 \lambda_1 + (1 - B_1 - B_2)] \\ & \quad + E_2 \lambda_2^{m-2} [B_2 \lambda_2^2 + B_1 \lambda_2 + (1 - B_1 - B_2)] \end{aligned} \quad (116)$$

The values of B_1 and B_2 may be determined for which the eigenvalues λ_1 and λ_2 are roots of the quadratic equation

$$B_2 \lambda^2 + B_1 \lambda + (1 - B_1 - B_2) = 0 \quad (117)$$

Then, from equation (116)

$$B_2 S_0 + B_1 S_1 + (1 - B_1 - B_2) S_2 \approx S_f \quad (118)$$

where the left side of the equation (118) converges to the exact solution.

Using equation (115) and eliminating terms containing E_2 :

$$\begin{aligned}(S_0 - S_1) - \lambda_2(S_1 - S_2) &= E_1 \lambda_1^{n-2}(\lambda_1 - \lambda_2)(\lambda_1 - 1) \\(S_1 - S_2) - \lambda_2(S_2 - S_3) &= E_1 \lambda_1^{n-3}(\lambda_1 - \lambda_2)(\lambda_1 - 1) \\(S_2 - S_3) - \lambda_2(S_3 - S_4) &= E_1 \lambda_1^{n-4}(\lambda_1 - \lambda_2)(\lambda_1 - 1)\end{aligned}\quad (119)$$

Solving for λ_1

$$\lambda_1 = \frac{(S_0 - S_1) - \lambda_2(S_1 - S_2)}{(S_1 - S_2) - \lambda_2(S_2 - S_3)} = \frac{(S_1 - S_2) - \lambda_2(S_2 - S_3)}{(S_2 - S_3) - \lambda_2(S_3 - S_4)} \quad (120)$$

From equations (117) and (120)

$$B_1 = \frac{(S_4 - S_3)(S_0 - 2S_2 + S_4) - (S_4 - S_2)[(S_1 - S_2) - (S_3 - S_4)]}{D} \quad (121)$$

$$B_2 = \frac{(S_4 - S_2)(S_4 - 2S_3 + S_2) - (S_4 - S_3)[(S_1 - S_2) - (S_3 - S_4)]}{D} \quad (122)$$

where $D = (S_4 - 2S_3 - S_2)(S_0 - 2S_2 + S_4) - (S_1 - S_2 - S_3 + S_4)^2$

The weighted averages of B_1 and B_2 are used for the extrapolation as done with A in equation (113). If the sum of the absolute values of the weighted averages of B_1 and B_2 changes by less than 2%, then the extrapolation is performed. Then from equation (118), the extrapolated solution is

$$S^* = \bar{B}_2 S_0 + \bar{B}_1 S_1 + (1 - \bar{B}_1 - \bar{B}_2) S_2 \quad (123)$$

10.0 CALCULATION OF VELOCITIES AND PRESSURE COEFFICIENTS

With the influence coefficients and the source densities determined, the calculation of velocities is a relatively simple matter. From equation (9), the total velocity is the sum of the freestream velocity and the disturbance velocity due to the body. The product of the source densities and the influence coefficients are summed for all of the elements, and then added to the freestream velocity to determine the total velocity at any point in the domain. Velocities on the surface of the body are calculated at the null points only, as the boundary conditions are enforced only at the null point of each element, and velocities at other points in the element would produce significant error due to the method of approximation. The components of the velocity at the centroid of the i th element are

$$\begin{aligned} V_{ix} &= V_{\infty x} + \sum_{j=1}^N C_{ijx} \sigma_j \\ V_{iy} &= V_{\infty y} + \sum_{j=1}^N C_{ijy} \sigma_j \\ V_{iz} &= V_{\infty z} + \sum_{j=1}^N C_{ijz} \sigma_j \end{aligned} \quad (124)$$

From equation (15), the velocity induced by an element at its own null point has a magnitude of 2π directed along the outward normal vector of the element.

At a point off the surface of the body, the components of the velocity are determined just as if the point of interest was a null point of a single element. The total velocity at the field point is the sum of the freestream velocity and the contributions of each of the elements of the body surface. The contribution of each of the elements is determined by calculating the influence coefficient based on the element geometry, and multiplying the result by the source density for the element. The total velocity at the field point may be expressed as

$$V_p = V_\infty + \sum_{q=1}^N C_{pq} \sigma_q \quad (125)$$

where p and q represent the field point and the source element respectively and the influence coefficient,

$$C_{pq} = \iint \frac{\partial}{\partial n} \left[\frac{1}{r(p,q)} \right] d\Gamma \quad (126)$$

As discussed in section 6.0, the influence coefficient may be calculated by the exact method, or it may be approximated by the quadrupole or monopole method depending on the ratio of the distance, r_0 , between the field point and the centroid of the source element to the maximum dimension, t , of the source element.

The magnitude of the velocity at either the on-body or off-body points is given by

$$|\mathbf{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2} \quad (127)$$

The pressure coefficient is calculated by using the result of equation (127) in equation (19), renumbered here for clarity.

$$C_p = \frac{p - p_\infty}{\frac{1}{2} \rho |\mathbf{v}_\infty|^2} = 1 - \frac{|\mathbf{v}|^2}{|\mathbf{v}_\infty|^2} \quad (128)$$

11.0 STREAMLINE CALCULATIONS

For the calculation of streamlines off the surface of the body, a timestep procedure is performed by calculating the velocity at the starting point of the streamline from equation (125), and advancing the streamline one time increment by a fourth order Runge-Kutta integration to a new point (Ralston 1965). The timestep procedure is repeated, thus creating a streamline composed of finite segments.

For the calculation of streamlines on the surface of the body, the streamline is started at a specified point and quadrilateral number. The local velocity is calculated from equation (124), and the values of a stream function are computed for each corner point. The stream function is chosen so that it has a value of zero at the last point on the streamline in the quadrilateral. The side of the quadrilateral through which the streamline exits is determined, and coordinates of the point on the side which has a stream function value of zero are computed. The direction of the streamline is verified by comparing it with the known direction of positive velocity. The next quadrilateral through which the streamline passes is determined by calculating the proximity of the new quadrilateral to the most recent point on the streamline. A circular area is computed which encloses the new quadrilateral with an additional 10% margin. If the last point of the streamline falls outside the circle, then the quadrilateral is discarded and a new one selected until the streamline is adjacent to the new quadrilateral.

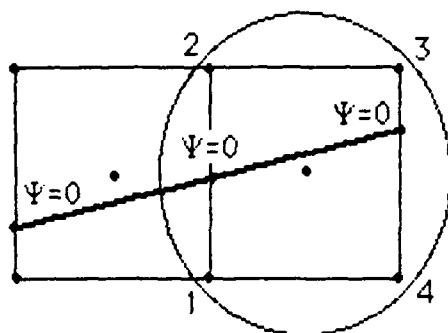


Figure 13. Calculation of on-body streamlines

This procedure is repeated along the surface of the body until all of the surface elements have been tested. The result is a streamline composed of segments from one side of an element to another.

12.0 DEVELOPMENT OF HIGHER ORDER PANEL METHODS

The XYZ Potential Flow program assumes a constant element source panel as described in Section 3.3. Extensive use of the constant element source panel method has shown that the primary disadvantage of the method is that, in order to obtain a highly accurate solution, a large number of surface elements must be used to discretize the body surface. The method has been applied to problems of increasingly complex configurations (Hess 1977). By doing so, the size of the coefficient matrix is increased resulting in increased computer time and cost. Additional cost is accrued due to the manhours required to prepare the input. Therefore, while the constant element methods have proven to be very successful, the cost has motivated the development of higher order methods.

The higher order surface singularity methods discretize the body surface with curved elements having a variable source density, as compared to the flat elements of constant source strength used in the basic method. Hess (1973) showed that the effect of a curved surface and the effect of a variable source density are of the same order of magnitude. Therefore, the two effects must be used together to provide a "consistent" solution. The consistent higher order panel method provides the increased accuracy and speed desired for three dimensional Neumann problems (Hess 1979).

According to Hess (1979), the evolution of the higher order panel

method from the constant element method involved the derivation of new influence coefficients based on the integration of a variable source density over a curved element. Other portions of the method were unchanged. However, the development of the higher order velocity equations also required different programming logic.

In examining the potential for development of the higher order methods, Hess (1979) noted that "a consistent approach always uses a source polynomial one degree less than the panel polynomial." Through an independent effort, Brebbia (1984) presented a higher order approach using the direct method to solve for a surface potential polynomial stating that the potential function must be of a degree at least equal to the degree of the polynomial describing the element. Knowing that the velocity function is the derivative of the potential function, these two observations agree. As a result of his derivations, Hess showed that the solution of a flat element with a constant source requires one integral, a paraboloidal panel with a linearly varying source density requires six integrals, and a cubic element with a quadratic source density requires twenty-three integrals. Development of higher order methods has focused on the paraboloidal element with the linearly varying surface, as solutions of higher order than that offered little benefit for the amount of effort required to produce a working program (Hess 1979). Hess (1979) and Eriksson (1983) have independently developed programs for three dimensional higher order panel methods. The higher order Hess program evolved from the constant element program which he developed in the early 1960s, while Eriksson developed a new program based on the

work of Johnson and Rubbert (1975). Continued work in the near future is expected to deal primarily with refinement of the paraboloidal element with a linearly varying source (Eriksson 1983).

In order to alleviate the burden of preparing the input, a geometry package for input data generation has been developed which is incorporated into the Hess higher order panel program. This allows the user to enter relatively few points to describe the body. The geometry package enhances the surface representation by distributing additional points on the surface based on one of many algorithms or recurring geometries (Halsey 1978).

As the state of the art in fluid dynamics has progressed, the XYZ Potential Flow program has seen increasingly complex applications requiring a great deal of effort in preparing the input, and requiring long computer run times. Hess (1979) reported the use of the Hess constant element program for a configuration utilizing 7000 effective elements. Realizing that the computation time increases as the square of the number of elements, it is easy to see the motivation for developing the higher order panel methods. Though modern computers offer storage capacities which can handle most applications of the constant element method, the higher order panel methods can provide equal accuracy for much less user effort. While the constant element method is still a versatile tool, future generations of the surface singularity methods will be able to handle the more complex applications being demanded in fluid dynamics.

13.0 VELOCITY CALCULATIONS FOR A TRIAXIAL ELLIPSOID

As the only true body for which an analytical solution exists, a triaxial ellipsoid was selected for the sample calculations in order to compare calculated results with the analytical solution. Hess has made use of the triaxial ellipsoid throughout his works in developing both the constant element method and the higher order panel method. Therefore, the XYZPF program will be compared with existing results of the Hess method (Hess 1979).

The triaxial ellipsoid utilized for the calculations has semiaxes dimensions of 1, 2, and 0.5 in the x, y, and z directions respectively. The surface was discretized by selecting fixed intervals of 0.1 in the y direction, and fourteen equal divisions of the 90° sector in the x-z plane. The values of x and z were then solved in terms of y and an angle θ . This method yielded 280 elements in the first octant for a total of 2240 effective elements after employing symmetry. A FORTRAN program was used to generate the corner points and to prepare the input file for later use by the XYZPF program.

Figures (16) and (17) show excellent correlation with the analytical solution and little difference from the Hess solution using 4320 effective elements. The use of the centroid as the control point is an approximation used to simplify the multipole expansion of the potential about the origin of the local coordinate system. This approximation is valid for most elements. However, for elements which are long and thin,

the physical difference between the centroid location and the null point location is significant, and use of the centroid can produce significant error as may be observed in figure (16) when the value of y approaches 2.0.

Recent calculations on the same body (Hess 1979) showed that results of at least equal accuracy could be obtained using only 480 effective elements using the higher order panel method. These results are a significant demonstration of the value of the higher order panel method. Using the higher order panel method rather than the constant element method, the user has the option of obtaining equal accuracy with cruder discretization or higher accuracy for the same discretization effort. While the results of the triaxial ellipsoid show relatively little improvement in accuracy, the most significant advantages of the higher order panel method are evident for a body with concave regions (Hess 1977).

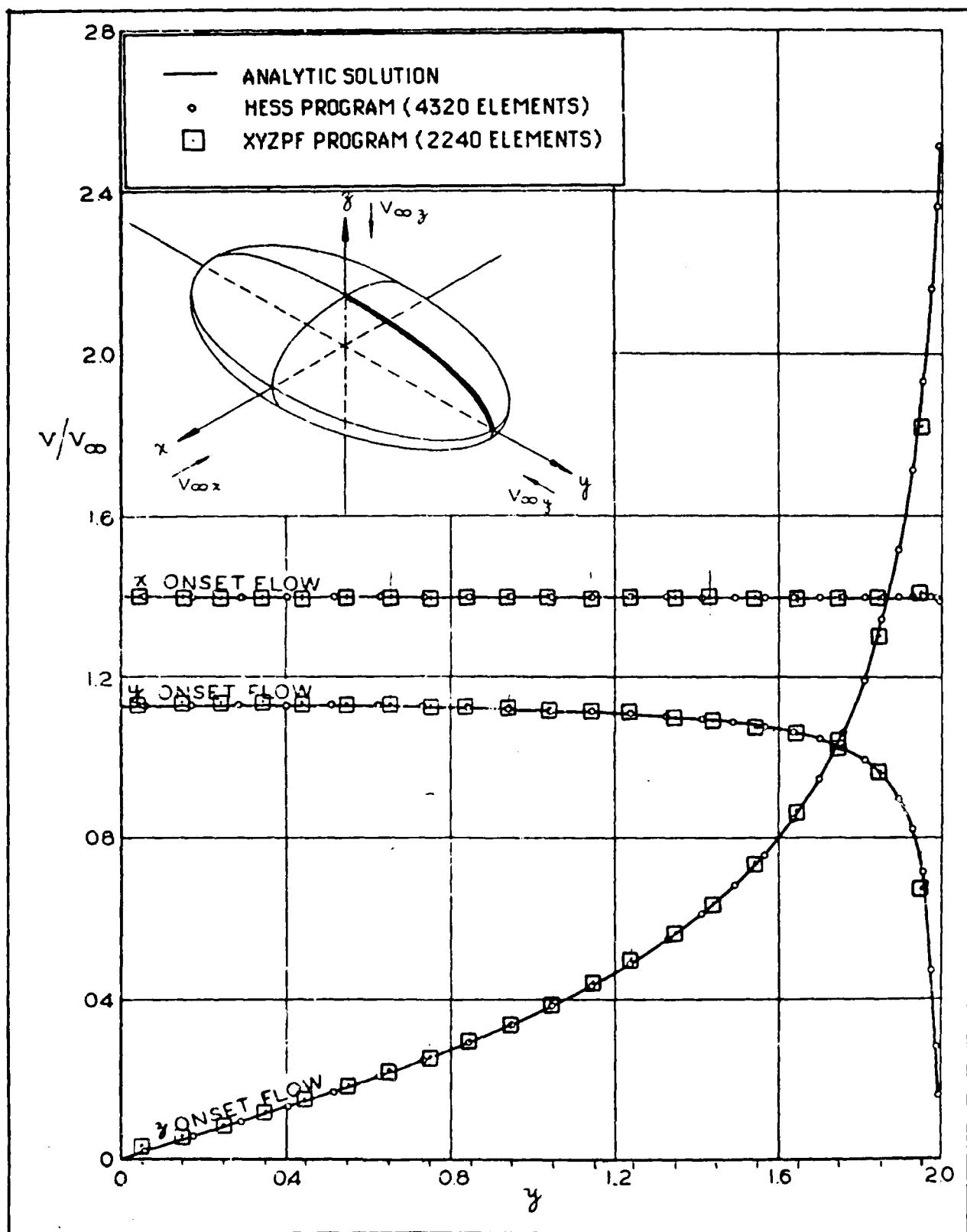


Figure 16. Comparison of analytic and calculated velocity distributions on an ellipsoid with axes ratios 1:2:0.5. Velocities in the xz-plane. (from Hess and Smith 1962)

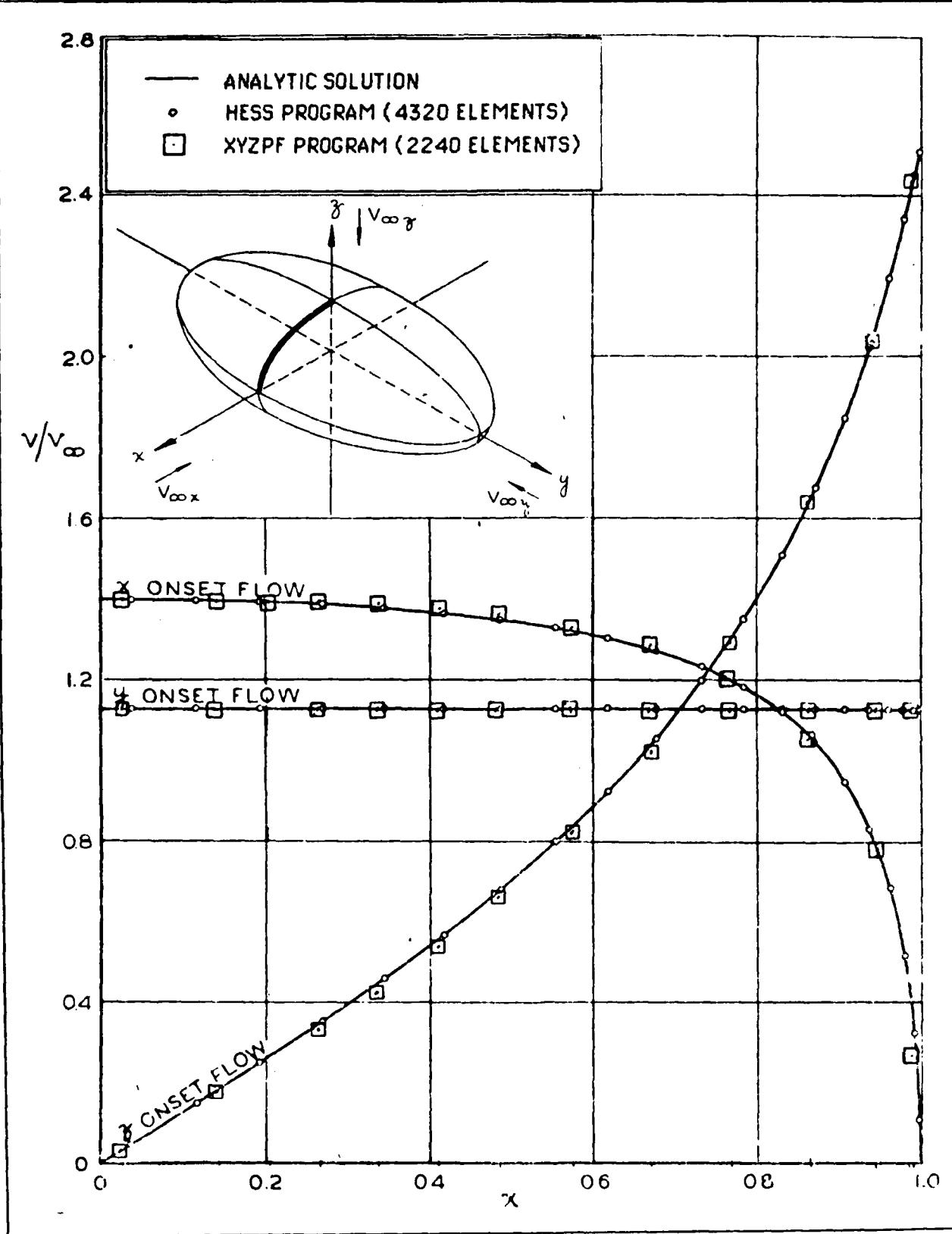


Figure 17. Comparison of analytic and calculated velocity distributions on an ellipsoid with axes ratios 1:2:0.5. Velocities in the yz -plane. (from Hess and Smith 1962)

14.0 CONCLUSION AND REMARKS

The objectives of this paper were (1) to describe the details of the approximation of an arbitrary three-dimensional body using quadrilateral elements, and (2) to provide a detailed derivation of the exact source panel integrations. Both of these objectives were met.

The method of surface discretization and source panel geometry is easily described using basic principles of geometry and vector algebra. By using quadrilateral surface elements, many surfaces can be discretized in a very straight forward logical fashion. The user can frequently visualize the contour lines of the surface which may be used to form the quadrilaterals, with some help from an intuitive approach to the fluid dynamics problem. The method of forming the planar quadrilateral element in the XYZPF Program differs slightly from the method presented by Hess and Smith (1962). The differences lie in the formation of the local coordinate system and the use of the centroid rather than the null point as the control point for applying the boundary conditions. The method of forming the local coordinate system has no effect on the potential flow calculations as long as one of the coordinate vectors is the outer normal to the planar element. The use of the centroid as the control point is an approximation used to simplify the multipole expansion of the potential about the origin of the local coordinate system. This approximation is valid for most elements. However, for elements which are long and thin, the physical difference between the centroid location and the null point location is significant, and use of the

centroid can produce significant error as may be observed in figure (16) when the value of y approaches 2.0.

A detailed derivation of the exact source panel integration has not previously appeared in literature, though the results are summarized by Hess and Smith (1962). The derivations presented in this paper verify the equations presented by Hess and Smith (1962), and the equations used in the XYZPF Program. The integral expressions for the velocity components were evaluated exactly with no assumptions or approximations used in the course of the integrations. Since the method of integration reduces the surface integral to a line integral around each of the sides of the element, the integration method can be generalized for a planar element with any number of sides, though the surface discretization used in the XYZPF Program uses only quadrilateral elements.

The calculation of potential flow about arbitrary three dimensional bodies is an engineering tool which is basic to design involving fluid dynamics. The XYZPF Program is a useful tool which has proven its value over the past 14 years. However, the increasing demands placed on this method are exposing the errors of the approximation as evident in the sample calculations presented in this paper. The requirement for increased accuracy has motivated the development of the higher order panel methods. Some of the limitations imposed on the XYZPF Program were due to computer memory and speed limitations. Advances in computer performance may allow future investigators to eliminate some

of the simplifying approximations used in the XYZPF Program, allowing increased accuracy without violating computer limitations. Some modifications might include the use of the null point as the control point rather than the panel centroid (as is used in the Hess program), or extending the range in which the exact velocity calculations are performed. The gains in accuracy by modifying the "constant element method" are limited by the basic approximations of the planar element and the constant source density for each element. Significant gains are most evident in the higher-order panel methods. This author concurs with Eriksson (1983) in expecting advances in the surface singularity methods to focus on the "development and refinement" of the higher order panel methods.

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APPENDIX I - XYZPF SECTION PF1

```
PROGRAM PPP1(INPUT=128,OUTPUT=128,TAPE5=INPUT,TAPE6=OUTPUT,TAPE03,
1           TAPE3=TAPE03,TAPE04,TAPE4=TAPE04,TAPE50=128)
C
C XYZ POTENTIAL FLOW PROGRAM VERSION 4 SECTION 1
C READS INPUT AND COMPUTES QUADRILATERAL PARAMETERS
C
C FOR INFORMATION CONTACT
C   BILL CHENG OR JANET DEAN
C   NUMERICAL FLUID DYNAMICS BRANCH CODE 1843
C   NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER
C   BETHESDA, MARYLAND 20084
C
C DIMENSION INDEX(9,3),G(9,6),F(9),C2(9),IP(9),XP(9),YP(9),ZF(9)
1           ,MSK(100), WS(240),PROB(15),DM(650)
COMMON      X(800 ),Y(800 ),Z(800 ),ID(41,71),B(250),T(4600),KP(100)
EQUIVALENCE (C2(1),F(1))
EQUIVALENCE(WS(1),KP(1)),(WS(101),MSK(1)),(WS(201),NP ),(WS(202),
2 NSP),(WS(203),NEP),(WS(204),NSE),(WS(205),MIX),(WS(206),MIY),
3 (WS(207),MIZ),(WS(209),ICUM),(WS(210),ISM),(WS(211),K),(WS(213),
4 EPS),(WS(208),IPS ),(WS(212),IPF ),(WS(217),XI ),(WS(218),YI ),
5 (WS(219),ZI )
EQUIVALENCE (Y12,Y23),(Y34,Y41)
INTEGER P,P1,P2,P3,P4,PC,P5,P6,P7,P8 ,P9
WRITE (6,5)
5 FORMAT(49H1XYZ POTENTIAL FLOW PROGRAM SECTION 1, VERSION 4 )
10 FORMAT (11,15R4)
20 FORMAT (1X,15I4)
50 FORMAT (1X,217,6E12.5)
30 FORMAT (1X,5F12.9)
C
C     A. READ IN CONTROL PARAMETERS
WS(220)=4.
K1=0
ID1=0
ID2=0
ID3=0
ID4=0
ID5=0
ID6=0
ID7=0
MAXN=70
MAXM=40
MAXNQE=650
MAXPC=800
ICNTRL=1
EOF50=0.
READ (5 ,10)J,(PROB(I),I=1,15)
IF (EOF(5) .EQ. 0.) GO TO 9
WRITE(6, 8)
8 FORMAT(39HNO TITLE CARD FOUND - PROGRAM ABORTED )
STOP
9 CONTINUE
J=0
WRITE (6,10) J,(PROB(I),I=1,15)
SA=.0
SB=.0.
21 FORMAT (17HNO. OF QUADS. =,14 /17H NO. OF SECTIONS=,
214/31H MAX. NO. OF ITERATIONS X FLOW ,13,9H Y FLOW ,
313,9H Z FLOW ,13)
READ (5 ,11)NQE,NSE,MIX,MIY,MIZ,ISM,EPS,IUCT,IPS,IPF,ISP
```

```

1 , IEDIT1, IEDIT3, IEDIT4, ITAPE, XCENTER, YCENTER, ZCENTER
11 FORMAT(6I4,F8.5,8I4,1X,3F5.3)
  IF (EOF(5) .EQ. 0.) GO TO 19
  WRITE(6,18)
18 FORMAT(43HONO PARAMETER CARD FOUND - PROGRAM ABORTED >
  STOP
19 CONTINUE
  IF (IEDIT1.EQ.1) ICNTRL=0
  WRITE(6,21) NQE, NSE, MIX, MIV, MIZ
31 FORMAT ("CONVERGENCE CRITERIA",F8.5)
41 FORMAT (4H0 M,7X,2HX1,12X,2HX2,12X,2HX3,12X,2HX4,12X,2HXP,12X,
  1 2HXR,12X,1HR,13X,3HCZ4/4H N,7X,2HY1,12X,2HY2,12X,2HY3,12X,2HY4,
  2 12X,2HYF,12X,2HYN,12X,2HFL,12X,3HCZ5/4H P,7X,2HZ1,12X,2HZ2,12X,
  3 2HZ3,12X,2HZ4,12X,2HZF,12X,2HZN,12X,4HCZ1 ,10X,3HCZ6/)
42 FORMAT (4H1 M,7X,2HX1,12X,2HX2,12X,2HX3,12X,2HX4,12X,2HXP,12X,
  1 2HXR,12X,1HR,13X,3HCZ4/4H N,7X,2HY1,12X,2HY2,12X,2HY3,12X,2HY4,
  2 12X,2HYF,12X,2HYN,12X,2HFL,12X,3HCZ5/4H P,7X,2HZ1,12X,2HZ2,12X,
  3 2HZ3,12X,2HZ4,12X,2HZP,12X,2HZN,12X,4HCZ1 ,10X,3HCZ6/)
24 FORMAT (1H ,11,19H PLANES OF SYMMETRY)
240 WRITE(6,24) ISM
270 WRITE(6,31) EPS
280 IF (IPS.LE.0) GO TO 290
285 WRITE(6,36) IPS,IPF
  36 FORMAT (45HNEW SOURCE DENSITY TO BE COMPUTED FOR QUADS.,14,3H - ,
  114)
290 K=0
  WRITE(6,39) ISP
39 FORMAT (9H0 ISP = ,13 )
  WRITE(6,37) IEDIT1, IEDIT3, IEDIT4, ITAPE
37 FORMAT (9H0IEDIT1 = ,13/9H IEDIT3 = ,13/9H IEDIT4 = ,13/9H ITAPE = ,
  1 13)
  WRITE(6,38) XCENTER, YCENTER, ZCENTER
38 FORMAT(10H0XCENTER = ,F5.2/10H YCENTER = ,F5.2/10H ZCENTER = ,F5.2)
  MM=0
  MN=0
  P=1
  Q=1.0
  DO 291 I=1,41
  DO 291 J=1,71
291 1D(I,J)=0
  J=0
C     B. READ FIRST PT.
  IERR=0
  IF (ITAPE.EQ.1) GO TO 292
2000 READ(5,40) XI,YI,ZI,N1,M1,NS,NE,UN
  IF (EOF(5).NE.0. OR. NS.LE.0) GO TO 2050
  WRITE(6,45) ICNTRL,NS
  45 FORMAT(11,9H SECTION ,14)
  LINE=0
  GO TO 293
2050 IF(IERR.EQ.0) GO TO 2200
  WRITE(6,2100)
2100 FORMAT(39HONO POINT CARDS FOUND - PROGRAM ABORTED >
  STOP
2200 IERR =1
  ITAPE=1
  WRITE(6,2300)
2300 FORMAT(47H0ERROR IN INPUT - POINT CARDS NOT ON INPUT FILE ,10X,
  1 53HPROGRAM WILL CHANGE ITAPE TO 1 AND TRY TO READ TAPE50 >
  IF (EOF(5).EQ.0) WRITE(6,2400) XI,YI,ZI
2400 FORMAT(11H0EXTRA FLOW,3F12.5,5X,20H11 WILL NOT BE COMPUTED >

```

```

292 READ(50,40) XI,YI,ZI,NI,MI,NS,NE,UN
40 FORMAT (3F12.9,4I4,F12.9)
EOF50=EOF(50)
IF (EOF50.NE.0. .OR. NS.LE.0) GO TO 2450
WRITE(6,45) ICNTRL,NS
LINE=0
GO TO 293
2450 IF (IERR.EQ.0) GO TO 2500
WRITE(6,2100)
STOP
2500 IERR=1
ITAPE=0
WRITE(6,2600)
2600 FORMAT("ERROR IN INPUT - POINT CARDS NOT ON TAPE50",10X,
1"PROGRAM WILL CHANGE ITAPE TO 0 AND TRY TO READ INPUT FILE")
GO TO 2000
293 UNR=UN
NSS=NS
PC=1
IF (NE.EQ.0) GO TO 2700
MMIN=NI
MMAX=NI
NMIN=MI
NMAX=MI
GO TO 300
2700 NMIN=MI
MMAX=MI
NMIN=NI
NMAX=NI
GO TO 300
295 IF (ITAPE.EQ.1) GO TO 297
READ (5,40) XI,YI,ZI,NI,MI,NS,ME,UN
IF (EOF(5).EQ.0.) GO TO 299
NS=0
XI=0.
YI=0.
ZI=0.
GO TO 299
297 READ(50,40) XI,YI,ZI,NI,MI,NS,ME,UN
EOF50=EOF(50)
IF (EOF50 .NE.0) NS=0
299 PC=PC+1
IF (NS.NE.NSS) GO TO 330
300 IF (NE.EQ.0) GO TO 304
301 IW=NI
NI=MI
MI=IW
C      C. STORE PT. IN PT. ARRAY
304 IF (MAXPC+1-PC) 295,305,310
305 WRITE(6,306) NS,MI,NI
306 FORMAT(60H ERROR IN INPUT - THERE ARE TOO MANY DATA POINTS IN SEC
ITION ,14,30H - POINTS BEGINNING WITH M = ,14,5H N = ,14,
2 17H WILL BE IGNORED )
LINE=LINE+1
ID4=ID4+1
GO TO 295
310 X(PC)=XI
Y(PC)=YI
Z(PC)=ZI
IF (MI.LE.MAXM .AND. NI.LE.MAXN) GO TO 315
WRITE(6,311) MI,NI

```

```

LINE=LINE+1
311 FORMAT(38H ERROR IN INPUT - INVALID M,N INDICES ,10X,
1 14HPOINT WITH M = ,14,5H N = ,14,17H WILL BE IGNORED >
105=105+1
PC=PC-1
GO TO 295
315 ID(M1,N1)=PC
MMAX=MAX0(MMAX,M1)
MMIN=MIND(MMIN,M1)
NMAX=MAX0(NMAX,N1)
NMIN=MIND(NMIN,N1)
GO TO 295
330 IF (IEDIT1.EQ.1) GO TO 294
IF (LINE.LT.40) GO TO 333
WRITE(6,42)
LINE=0
GO TO 294
333 WRITE(6,41)
294 CONTINUE
C      E. DO LOOPS TO SWEEP PT. ARRAY
N1=NMIN
M12=MMAX-MMIN
NN2=NMAX-NMIN
IF ( MOD(MM2,2).EQ.0 .AND. MOD(NN2,2).EQ.0 ) GO TO 332
WRITE(6,331) N35,MMIN,MMAX,MMIN,NN2
LINE=LINE+1
331 FORMAT(16HERROR - SECTION ,15,45H DOES NOT HAVE QUADS ARRANGED IN
1 BLOCKS OF 4 ,9H      MMIN= ,12,6H MMAX= ,12,6H NMIN= ,12 ,
26H NMAX= ,12)
1D6=1D6+1
332 MM2=MM2/2
NN2=NN2/2
DO 404 NH=1,NN2
M1=MMIN
DO 402 MM=1,MM2
NQ=1
C      F. HAVE 9 CORNER PTS. BEEN GIVEN
IT=ID(M1,N1)*ID(M1+1,N1)*ID(M1+2,N1)*ID(M1,N1+1)*ID(M1+1,N1+1)*
1 ID(M1+1,N1+2)*ID(M1,N1+2)*ID(M1+1,N1+2)*ID(M1+2,N1+2)
IFC IT.EQ.0 > GO TO 402
IERR=0
M2=M1+1
N2=N1+1 =M1,M2
DO 400 N=N1,N2
GO TO (334,335,336,337) NO
334 P1=ID(M ,N )
P2=ID(M+1,N )
P3=ID(M+1,N+1)
P4=ID(M ,N+1)
P5=ID(M+2,N )
P6=ID(M+2,N+1)
P7=ID(M+1,N+2)
P8=ID(M ,N+2)
P9=P1
IF((X(P1).NE.X(P2)).OR.(Y(P1).NE.Y(P2)).OR.(Z(P1).NE.Z(P2))) .AND.
1 ((X(P1).NE.X(P4)).OR.(Y(P1).NE.Y(P4)).OR.(Z(P1).NE.Z(P4))) GO TO 340
P9=ID(M+2,N+2)
GO TO 340
335 P1=ID(M ,N+1)
P2=ID(M ,N )
P3=ID(M+1,N )

```

```

P4=ID(M+1,N+1)
P5=ID(M ,N-1)
P6=ID(M+1,N-1)
P7=ID(M+2,N )
P8=ID(M+2,N+1)
P9=P1
IF((X(P1).NE.X(P2).OR.Y(P1).NE.Y(P2).OR.Z(P1).NE.Z(P2)) .AND.
1 (X(P1).NE.X(P4).OR.Y(P1).NE.Y(P4).OR.Z(P1).NE.Z(P4))) GO TO 340
P9=ID(M+2,N-1)
GO TO 340
336 P1=ID(M+1,N )
P2=ID(M+1,N+1)
P3=ID(M ,N+1)
P4=ID(M ,N )
P5=ID(M+1,N+2)
P6=ID(M ,N+2)
P7=ID(M-1,N+1)
P8=ID(M-1,N )
P9=P1
IF((X(P1).NE.X(P2).OR.Y(P1).NE.Y(P2).OR.Z(P1).NE.Z(P2)) .AND.
1 (X(P1).NE.X(P4).OR.Y(P1).NE.Y(P4).OR.Z(P1).NE.Z(P4))) GO TO 340
P9=ID(M-1,N+2)
GO TO 340
337 P1=ID(M+1,N+1)
P2=ID(M ,N+1)
P3=ID(M ,N )
P4=ID(M+1,N )
P5=ID(M-1,N+1)
P6=ID(M-1,N )
P7=ID(M ,N-1)
P8=ID(M+1,N-1)
P9=P1
IF((X(P1).NE.X(P2).OR.Y(P1).NE.Y(P2).OR.Z(P1).NE.Z(P2)) .AND.
1 (X(P1).NE.X(P4).OR.Y(P1).NE.Y(P4).OR.Z(P1).NE.Z(P4))) GO TO 340
P9=ID(M-1,N-1)
340 IP(1)=P1
IP(2)=P2
IP(3)=P3
IP(4)=P4
IP(5)=P5
IP(6)=P6
IP(7)=P7
IP(8)=P8
IP(9)=P9
C      G2 COMPUTE NORMAL VECTOR (XN,YN,ZN)
X1=X(P3)-X(P1)
X2=X(P4)-X(P2)
Y1=Y(P3)-Y(P1)
Y2=Y(P4)-Y(P2)
Z1=Z(P3)-Z(P1)
Z2=Z(P4)-Z(P2)
XN=Y2*Z1-Y1*Z2
YN=X1*Z2-X2*Z1
ZN=X2*Y1-X1*Y2
R=S02(XN,YN,ZN)
IF (R.GT. .00000000001) GO TO 345
WRITE(6,343)
343 FORMAT(33H ERROR IN INPUT - ZERO AREA QUAD >
LINE=LINE+1
ID1=ID1+1
AQ=0.

```

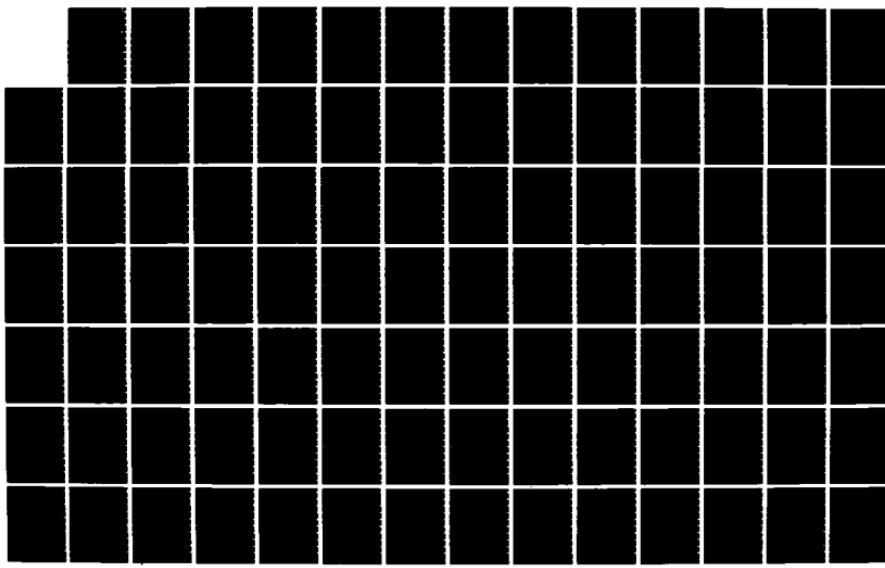
AD-A168 167

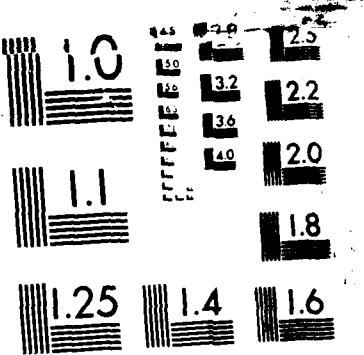
FORMULATION OF NUMERICAL METHODS USED IN THE XYZ
THREE-DIMENSIONAL POTENT. (U) TEXAS A AND M UNIV
COLLEGE STATION COLL OF ENGINEERING W J BEARY MAY 86
UNCLASSIFIED N00228-85-G-3203

2/3

F/G 20/4

NL





MICROCOPY RESOLUTION TEST CHART

```

XC=0.
YC=0.
ZC=0.
FL=0.
CZ(1)=0.
CZ(4)=0.
CZ(5)=0.
CZ(6)=0.
IERR=1
GO TO 351
345 CONTINUE
XN=XN/R
YN=YN/R
ZN=ZN/R
A0=.5*R
C COMPUTE CENTROID
X1=X(P3)-X(P2)
Y1=Y(P3)-Y(P2)
Z1=Z(P3)-Z(P2)
X5=Y1*Z2-Y2*Z1
Y5=Z1*X2-Z2*X1
Z5=X1*Y2-X2*Y1
A1=SQ2(X5,Y5,Z5)
A2=R-A1
IT=1
XC=(X(P2)+X(P4)+(A1*X(P3)+A2*X(P1))/R)/3.
YC=(Y(P2)+Y(P4)+(A1*Y(P3)+A2*Y(P1))/R)/3.
ZC=(Z(P2)+Z(P4)+(A1*Z(P3)+A2*Z(P1))/R)/3.
C COMPUTE SECOND AND THIRD VECTORS
945 X4=YN*Z1-Y1*ZN
Y4=ZN*X1-Z1*XN
Z4=XN*Y1-X1*YN
R=1./SQ2(X4,Y4,Z4)
X4=X4*R
Y4=Y4*R
Z4=Z4*R
X3=ZN*Y4-Z4*YN
Y3=XN*Z4-X4*ZN
Z3=YN*X4-Y4*XN
C COMPUTE POINTS IN QUAD SYSTEM
DO 947 I=1,9
L=IP(I)
XP(I)=X3*(X(L)-XC)+Y3*(Y(L)-YC)+Z3*(Z(L)-ZC)
YP(I)=X4*(X(L)-XC)+Y4*(Y(L)-YC)+Z4*(Z(L)-ZC)
947 ZP(I)=XN*(X(L)-XC)+YN*(Y(L)-YC)+ZN*(Z(L)-ZC)
C COMPUTE MATRIX COEF. TO FIND SURFACE EQ.
DO 949 I=2,9
G(I,1)=1.
G(I,2)=XP(I)
G(I,3)=YP(I)
G(I,4)=XP(I)**2
G(I,5)=YP(I)**2
G(I,6)=YP(I)*XP(I)
949 F(I)=ZP(I)
DO 953 I=1,6
G(1,I)=G(9,I)
G(5,I)=G(5,I)+G(6,I)
953 G(6,I)=G(7,I)+G(8,I)
F(1)=F(9)
F(5)=F(5)+F(6)
F(6)=F(7)+F(8)

```

```

C      SOLVE MATRIX EQ. G*CZ=F FOR CZ
CALL MATINS(G,0,6,F,6,1,DETERM,1DM,INDEX)
IF (1DM.EQ. 1) GO TO (955,960) IT
IERR=1
WRITE(6,954)
954 FORMAT (33H ERROR IN INPUT - SINGULAR MATRIX )
LINE=LINE+1
ID2=ID2+1
GO TO 960
955 IT=2
C      FIND NEW NORMAL VECTOR
XN=XN-CZ(2)*X3-CZ(3)*X4
YN=YN-CZ(2)*Y3-CZ(3)*Y4
ZN=ZN-CZ(2)*Z3-CZ(3)*Z4
A=1./SQ2(XN,YN,ZN)
XN=XN*A
YN=YN*A
ZN=ZN*A
GO TO 945
C      STORE DATA
960 B(J+1)=XP(1)
B(J+2)=YP(1)
B(J+3)=XP(2)
B(J+4)=YP(2)
B(J+5)=XP(3)
B(J+6)=XP(4)
B(J+7)=YP(4)
B(J+8)=X3
B(J+9)=Y3
B(J+10)=Z3
B(J+11)=X4
B(J+12)=Y4
B(J+13)=Z4
B(J+14)=CZ(1)
B(J+15)=CZ(4)
B(J+16)=CZ(5)
B(J+17)=CZ(6)
IF (K .LT. 7*MAXNOE) GO TO 965
ID7=ID7+1
K1=K+K1
K=0
965 CONTINUE
T(K+1)=XC
T(K+2)=YC
T(K+3)=ZC
T(K+4)=XN
T(K+5)=YN
T(K+6)=ZN
T(K+7)=RQ
C      COMPUTE QUADRUPOLE MOMENTS
X11=XP(1)+XP(2)
X12=XP(1)+XP(4)
X13=XP(3)+XP(2)
X14=XP(3)+XP(4)
X15=XP(2)+XP(4)
Y11=YP(1)+YP(2)
Y12=YP(1)+YP(4)
Y13=YP(3)+YP(2)
Y14=YP(3)+YP(4)
Y15=YP(2)+YP(4)
R1=R1/24.

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```

R2=R2/24.
R3=RAQ/12.
RXX=(X11**2+X12**2)*R1+(X13**2+X14**2)*R2+X15**2*R3
RXY=(X11*V11+X12*V12)*R1+(X13*V13+X14*V14)*R2+X15*V15*R3
RYY=(Y11**2+Y12**2)*R1+(Y13**2+Y14**2)*R2+Y15**2*R3
C COMPUTE SOLID ANGLE
XX=XC-XCENTER
YY=YC-YCENTER
ZZ=ZC-ZCENTER
X1=XX*X3+YY*Y3+ZZ*Z3
Y1=XX*X4+YY*Y4+ZZ*Z4
Z1=XX*XN+YY*YN+ZZ*ZN
RD=1./SQ2(X1,Y1,Z1)
RCU=RD**3
RSU=RCU**2*RD
SA=SA+Z1*(AQ*RCU-(RXX*(Y1**2+Z1**2-4.*X1**2)
1 +RXY*(X1**2+Z1**2-4.*Y1**2))*1.5-15.*X1*Y1*RXY)*RSU)
B(J+18)=RXX
B(J+19)=RXY
B(J+20)=RYY
C ERROR TESTS
D1=SQ2((XP(3)-XP(1)),(YP(3)-YP(1)),0.)
D2=SQ2((XP(4)-XP(2)),(YP(4)-YP(2)),0.)
FL=.5*AMAX1(D1,D2)
CZ23=ABS(CZ(2))+ABS(CZ(3))
IF(ABS(CZ(2))+ABS(CZ(3)) .GT. FL*.001 ) GO TO 970
IF( ABS(CZ(1)) .LT. FL*.3 ) GO TO 977
970 WRITE(6,975) CZ23
975 FORMAT(29H QUESTIONABLE POINT -POOR FIT ,6E14.3)
IERR=1
LINE=LINE+1
977 IF (YP(1)*XP(2)-YP(2)*XP(1) .GE. 0. .AND.
1 YP(2)*XP(3)-YP(3)*XP(2) .GE. 0. .AND.
2 YP(3)*XP(4)-YP(4)*XP(3) .GE. 0. .AND.
3 YP(4)*XP(1)-YP(1)*XP(4) .GE. 0.) GO TO 984
980 WRITE(6,1000) (XP(I),YP(I), I=1,4)
1000 FORMAT(41H ERROR IN INPUT - CROSSED OR CONCAVE QUAD , ,
1 4(2F10.5,3X) )
IERR=1
LINE=LINE+1
ID3=ID3+1
984 CRCF=SQ2((XP(2)-XP(1)),(YP(2)-YP(1)),0)+XP(3)-XP(2)+ SQ2((XP(1)-
1 XP(4)),(YP(1)-YP(4)),0.)+SQ2((XP(4)-XP(3)),(YP(4)-YP(3)),0.)
IF (.36.*AQ .GT. CRCF**2 ) GO TO 986
LINE=LINE+1
WRITE(6,951)
961 FORMAT(24H WARNING LONG THIN QUAD.)
986 IF ( Z1 .GE. 0. ) GO TO 351
350 WRITE(6,35)
85 FORMAT(35H QUESTIONABLE POINT - INWARD NORMAL)
IERR=1
LINE=LINE+1
C J. EDIT QUAD INFORMATION
351 IF (IEDIT1.EQ.2 .AND. IERR.EQ.0) GO TO 354
IF (IEDIT1.EQ.1 ) GO TO 354
GO TO (356,357,358,359) NO
356 WRITE(6,51) M,X(P1),X(P2),X(P3),X(P4),YC,XN,AQ ,CZ(4) ,
1 N,Y(P1),Y(P2),Y(P3),Y(P4),YC,YN,FL ,CZ(5) ,
2 P,Z(P1),Z(P2),Z(P3),Z(P4),ZC,ZN,CZ(1),CZ(6)
GO TO 360
357 WRITE(6,51) M,X(P2),X(P3),X(P4),X(P1),YC,XN,AQ ,CZ(4) ,

```

```

1           N,Y(P2),Y(P3),Y(P4),Y(P1),YC,YN,FL   ,CZ(5)  ,
2           P,Z(P2),Z(P3),Z(P4),Z(P1),ZC,ZN,CZ(1),CZ(6)
GO TO 360
358 WRITE(6,51) M,X(P4),X(P1),X(P2),X(P3),XC,XN,RQ   ,CZ(4)  ,
1           N,Y(P4),Y(P1),Y(P2),Y(P3),YC,YN,FL   ,CZ(5)  ,
2           P,Z(P4),Z(P1),Z(P2),Z(P3),ZC,ZN,CZ(1),CZ(6)
GO TO 360
359 WRITE(6,51) M,X(P3),X(P4),X(P1),X(P2),XC,XN,RQ   ,CZ(4)  ,
1           N,Y(P3),Y(P4),Y(P1),Y(P2),YC,YN,FL   ,CZ(5)  ,
2           P,Z(P3),Z(P4),Z(P1),Z(P2),ZC,ZN,CZ(1),CZ(6)
360 CONTINUE
51 FORMAT (1H ,I3,8E14.5/1X,I3,8E14.5/1X,I3,8E14.5/)
LINE=LINE+4
IF (LINE.LT.50) GO TO 354
352 WRITE (6,42)
LINE=0
354 CONTINUE
J=J+20
I=P
DM(I)=UNR
P=P+1
NQ=NQ+1
IERR=0
349 K=K+7
C      K.  WRITE OUT BLOCK OF B ARRAY IF FULL
IF (J.LT.240) GO TO 400
355 WRITE (04) Q,(B(I),I=1,240)
Q=P
J=0
C      L.  END OF DO LOOP OVER PT. ARRAY
400 CONTINUE
402 M1=M1+2
404 N1=N1+2
C      M.  SET FOR NEXT SECTION
NSS=NS
DO 405 M=MMIN,MMAX
DO 405 N=NMIN,NMAX
405 ID(M,N)=0
PC=1
IF (ME .EQ. 0) GO TO 410
MMAX=N1
MMIN=N1
NMIN=M1
NMAX=M1
GO TO 420
410 MMAX=M1
MMIN=M1
NMIN=N1
NMAX=N1
420 NE=ME
UNR=UN
IF (NS.LE.0) GO TO 500
WRITE(6,45) ICNTRL,NS
LINE=0
GO TO 300
500 WRITE (04) Q,(B(I),I=1,240)
550 NP=(K+K1)/?
ISM = ISM + 1
GO TO (595,590,580,570),ISM
570 SA=SA+SA
580 SA=SA+SA

```

```

590 SA=SA+SA
C      01 WRITE PARAMETERS AND T ARRAY ON TAPE
595 J = 1
      IF (ITAPE.EQ.1 .AND. EOF50 .NE. 0)      GO TO 601
597 WS(J) = XI
      WS(J+20) = YI
      WS(J+40) = ZI
      J = J+1
      IF(XI**2 + YI**2 + ZI**2) 599,599,598
598 WRITE(6,600) XI,YI,ZI
600 FORMAT(11H0EXTRA FLOW,10X,3F12.5)
601 READ(5,40) XI,YI,ZI
      IF (EOF(5) .EQ. 0) GO TO 597
      XI=0.
      YI=0.
      ZI=0.
      GO TO 597
599 IF (ISP.LT.0) GO TO 605
      WRITE (03) (PROB(I),I=1,15)
      WRITE (03) (WS(I),I=1,220),IEDIT3,IEDIT4
      WRITE (03) (T(I),I=1,K)
      WRITE (03) (DM(I),I=1,NP)
605 CONTINUE
C      N1 CHECK SOLID ANGLE
610 WRITE (6,80) SA
60 FORMAT(14H0SOLID ANGLE = ,F8.3)
620 REWIND 04
REWIND 03
622 I0S=I01+I02+I03+I04+I05+I06+I07
      IF (I0S.EQ.0 .AND. NP.EQ.NQE) GO TO 638
      WRITE(6,625)
      IF (I01.GT.0) WRITE(6,628) I01
      IF (I02.GT.0) WRITE(6,629) I02
      IF (I03.GT.0) WRITE(6,630) I03
      IF (I04.GT.0) WRITE(6,631) I04
      IF (I05.GT.0) WRITE(6,632) I05
      IF (I06.GT.0) WRITE(6,633) I06
      IF (I07.GT.0) WRITE(6,634) NP,MAXNQE
      IF (NP.NE.NQE) WRITE(6,637) NP,NQE
      STOP
625 FORMAT(3SHOFATAL ERROR IN DATA - PROGRAM ABORTED)
628 FORMAT(1H0,15,31H QUADRILATERALS WITH ZERO AREA          )
629 FORMAT(1H0,15,44H QUADRILATERALS GENERATE A SINGULAR MATRIX   )
630 FORMAT(1H0,15,26H CROSSED QUADRILATERALS          )
631 FORMAT(1H0,15,32H SECTIONS HAVE TOO MANY POINTS    )
632 FORMAT(1H0,15,34H POINTS HAVE INVALID M,N INDICES   )
633 FORMAT(1H0,15,52H SECTIONS DO NOT HAVE QUADS ARRANGED IN GROUPS 0
      1F 4  )
634 FORMAT(1H0,15,48H QUADRILATERALS GIVEN, EXCEEDING THE LIMIT OF  ,
      1 14)
637 FORMAT(1H0,15,28H QUADRILATERALS GIVEN, NOT  ,14)
638 IF (ISP.LE.0) GO TO 640
      WRITE(6,639) ISP
639 FORMAT(7H0 ISP= ,14,19H - PROGRAM ABORTED   )
      STOP
640 CONTINUE
C      02 READ PEPS2 AND TRANSFER TO IT
      STOP 1
      END
C      FUNCTION SQ2(X,Y,Z)

```

```

C COMPUTE SQUAR ROOT OF R**2
R= ABS(X)+ABS(Y) +ABS(Z) +.0000000000001
700 RS=X**2+Y**2 +Z**2
R=R+RS/R
R=.25*R+RS/R
R=R+RS/R
SQ2=.25*R+RS/R
RETURN
END
C
C SUBROUTINE MATINS(A,NR,N1,B,NC,M1,DETERM, ID, INDEX)
C MATRIX INVERSION WITH ACCOMPANYING SOLUTION OF SIMUL. EQ.
C PIVOT METHOD
C FORTRAN IV SINGLE PRECISION WITH ADJUSTABLE DIMENSION
C NOVEMBER 1971 S GOOD NAVAL SHIP R & D CENTER
C WHERE CALLING PROGRAM MUST INCLUDE
C           DIMENSION A(NR,NR), B(NR,NC), INDEX(NR,3)
C           WHERE NR,NC ARE DIMENSIONS OF A,B, INDEX
C           N1 IS THE ORDER OF A
C           M1 IS THE NUMBER OF COLUMN VECTORS IN B (MAY BE 0)
C           DETERM WILL CONTAIN DETERMINANT ON EXIT
C           ID WILL BE SET BY ROUTINE TO 2 IF MATRIX A IS
C           SINGULAR, 1 IF INVERSION WAS SUCCESSFUL
C           MATRIX A (INPUT MATRIX) WILL BE REPLACED BY A INV
C           MATRIX B: THE COLUMN VECTORS WILL BE REPLACED
C           BY CORRESPONDING SOLUTION VECTORS
C           INDEX: WORKING STORAGE ARRAY
C           IF IT IS DESIRED TO SCALE, THE DETERMINANT CARD 29 MAY BE
C           DELETED AND DETERM PRESSET BEFORE ENTERING THE ROUTINE
C
C           DIMENSION A(NR,NR), B(NR,NC), INDEX(NR,3)
C           EQUIVALENCE (IROW,JROW), (ICOLUMN,JCOLUMN), (AMAX, T, SWAP)
C
C           INITIALIZATION
C
N=N1
M=M1
DETERM=1.0
DO 20 J=1,N
C
20 INDEX(J,3)=0
DO 550 I=1,N
C
C           SEARCH FOR PIVOT ELEMENT
C
AMAX = 0.0
DO 105 J=1,N
IF(INDEX(J,3)-1) 60, 105, 60
60 DO 100 K=1,N
IF(INDEX(K,3)-1) 80, 100, 715
80 IF (AMAX -ABS (A(J,K))) 85, 100, 100
85 IROW = J
ICOLUMN = K
AMAX = ABS (A(J,K))
100 CONTINUE
105 CONTINUE
INDEX(ICOLUMN,3) = INDEX(ICOLUMN,3) + 1
INDEX(1,1) = IROW
INDEX(1,2) = ICOLUMN
C
C           INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL

```

```

C
  IF (IROW-ICOLUMN) 140, 310, 140
140 DETERM= -DETERM
  DO 200 L=1,N
    SWAP= A(IROW,L)
    A(IROW,L)=A(ICOLUMN,L)
200 A(ICOLUMN,L)=SWAP
  IF(M) 310, 310, 210
210 DO 250 L=1,M
    SWAP=B(IROW,L)
    B(IROW,L)=B(ICOLUMN,L)
250 B(ICOLUMN,L)=SWAP
C
C      DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
  310 PIVOT = A(ICOLUMN,ICOLUMN)
  DETERM=DETERM*PIVOT
  330 A(ICOLUMN,ICOLUMN) = 1.0
  DO 350 L=1,N
  350 A(ICOLUMN,L)=A(ICOLUMN,L)/PIVOT
  IF (M) 380, 380, 360
  360 DO 370 L=1,M
  370 B(ICOLUMN,L)=B(ICOLUMN,L)/PIVOT
C
C      REDUCE NON-PIVOT ROWS
C
  380 DO 550 L1=1,N
    IF(L1-ICOLUMN) 400, 550, 400
  400 T=A(L1,ICOLUMN)
    A(L1,ICOLUMN)=0.0
    DO 450 L=1,N
  450 A(L1,L)=A(L1,L)-A(ICOLUMN,L)*T
    IF (M) 550, 550, 460
  460 DO 500 L=1,M
  500 B(L1,L)=B(L1,L)-B(ICOLUMN,L)*T
  550 CONTINUE
C
C      INTERCHANGE COLUMNS
C
  DO 710 I=1,N
    L=N+1-I
    IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630
  630 JROW= INDEX(L,1)
    JCOLUMN=INDEX(L,2)
    DO 705 K=1,N
      SWAP = A(K,JROW)
      A(K,JROW)=A(K,JCOLUMN)
      A(K,JCOLUMN)=SWAP
  705 CONTINUE
  710 CONTINUE
  DO 730 K=1,N
    IF(INDEX(K,3)-1) 715, 720, 715
  720 CONTINUE
  730 CONTINUE
    ID=1
  810 RETURN
  715 ID=2
  GO TO 810
END
/

```

APPENDIX II - XYZPF SECTION PF2

```
PROGRAM PF2

C
C   XYZ POTENTIAL FLOW PROGRAM  VERSION 4  SECTION 2
C   COMPUTES MATRIX COEFFICIENTS
C

COMMON      B(241),T(4600),U1(1000),U2(1000),U3(1000),C1( 900),C2( 9
200),C3( 900),
3UX(8),UY(8),UZ(8)
EQUIVALENCE(Y3,Y2)      ,(WS(201),NP),(WS(210),SYM),(WS(211),KM)
1     ,(WS(212),IPF)    ,(WS(208),IPS) ,(KM,MK)
INTEGER SYM
BLK=1.0
IDW=0
READ(03)    (PROB(I),I=1,15)
WRITE(6,5)
5 FORMAT(49HXYZ POTENTIAL FLOW PROGRAM SECTION 2, VERSION 4  )
90 FORMAT(1HD,15A4)
      WRITE(6,90)          (PROB(I),I=1,15)
C       A.  READ PARAMETERS, T ARRAY, FIRST BLOCK OF B ARRAY
READ(03)    (WS(I),I=1,220)
READ(03)    (T(I),I=1,MK)
READ(04)    (B(I),I=1,241)
C       B.  START LOOP OVER QUADRILATERALS
K=1
J=1
P=1
JC=1
JU=2
NPN=5*((NP+4)/5)
KMM=7+NPN+1
290 IF(B(1)-P>595,295,595
98 FORMAT("POINTS OUT OF ORDER  B(1)=",1F4.0," P=",1F4.0)
295 J=2
296 X1=B(J)
Y1=B(J+1)
X2=B(J+2)
Y2=B(J+3)
Y3=B(J+4)
C
Y3=Y2
Y4=B(J+5)
Y4=B(J+6)
XN=T(K+3)
YN=T(K+4)
ZN=T(K+5)
XC=T(K+6)
YC=T(K+7)
ZC=T(K+8)
R=T(K+9)
XX=B(J+10)
YX=B(J+11)
ZX=B(J+12)
XY=B(J+13)
YY=B(J+14)
ZY=B(J+15)
C1  COMPUTE LENGTH OF SIDES OF QUAD
D12=S02F(X1,X2,Y1,Y2,0.,0.)
D23=S02F(X2,X3,Y2,Y3,.0.,0.)
```

```

D34=SQ2F(X3,X4,Y3,Y4,.0,.0)
D41=SQ2F(X4,X1,Y4,Y1,.0,.0)
C      C2 COMPUTE SLOPE OF SIDES
IF(X2-X3)305,300,305
300 C123=1.
GO TO 310
305 CM23=(Y2-Y3)/(X2-X3)
C123=0.
310 IF(X3-X4)315,311,315
311 C134=1.
GO TO 320
315 CM34=(Y4-Y3)/(X4-X3)
C134=0.
320 IF(X4-X1)325,321,325
321 C141=1.
GO TO 330
325 CM41=(Y1-Y4)/(X1-X4)
C141=0.
330 IF(X1-X2)335,331,335
331 C112=1.
GO TO 340
335 CM12=(Y2-Y1)/(X2-X1)
C112=0.
C      C3 COMPUTE QUADRAPOLE MOMENTS
340 C1XX=B(J+17)
C1XY=B(J+18)
C1YY=B(J+19)
CY12=0.0
CX12=0.0
CY23=0.0
CX23=0.0
CY34=0.0
CX34=0.0
CY41=0.0
CX41=0.0
C      C4 COMPUTE SIN AND COS OF SLOPE ANGLE FOR EACH SIDE
IF(D12) 9341,9342,9341
9341 CY12=(Y2-Y1)/D12
CX12=(X1-X2)/D12
9342 IF(D23)9343,9344,9343
9343 CY23=(Y3-Y2)/D23
CX23=(X2-X3)/D23
9344 IF(D34)9345,9346,9345
9345 CY34=(Y4-Y3)/D34
CX34=(X3-X4)/D34
9346 IF(D41)9347,9348,9347
9347 CY41=(Y1-Y4)/D41
CX41=(X4-X1)/D41
C      C5 COMPUTE MAX LENGTH OF QUAD
9348 ST=RE5(X1-X3)
ST2=SQ2F(X2,X4,Y2,Y4,.0,.0)
ST=RMAX1(ST,ST2,D12,D23,D34,D41)
C      D. START LOOP OVER NULL PTS
342 K0=1
L=1
343 I=K0
IF(IPS) 9360,9360,9350
9350 IF(L-1PS) 9355,9352,9352
9352 IF(L-1PF) 9360,9360,9355
9355 C1(JC)=.0
C2(JC)=.0
C3(JC)=.0

```

GO TO 541
 9360 IS=1
 XCQ=T(1)
 YCQ=T(1+1)
 ZCQ=T(1+2)
 XNQ=T(1+3)
 YNQ=T(1+4)
 344 ZNQ=T(1+5)
 C E. COMPUTE DISTANCE BETWEEN QUAD AND NULL PT.
 C . DETERMIN METHOD
 345 RPQ=SQ2F(XC,XCQ,YC,YCQ,ZC,ZCQ)
 IF(RPQ-ST*4)350,350,460
 350 X=(XCQ-XC)*XX+(YCQ-YC)*YX+(ZCQ-ZC)*ZX
 Y=(XCQ-XC)*XY+(YCQ-YC)*YY+(ZCQ-ZC)*ZY
 Z=(XCQ-XC)*XN+(YCQ-YC)*YN+(ZCQ-ZC)*ZN
 IF(RPQ-ST*2.0)355,355,400
 C F. COMPUTE VELOCITY COEF. BY EXACT METHOD
 355 R1=SQ2F(X,X1,Y,Y1,Z,0.)
 R2=SQ2F(X,X2,Y,Y2,Z,0.)
 R3=SQ2F(X,X3,Y,Y3,Z,0.)
 R4=SQ2F(X,X4,Y,Y4,Z,0.)
 IF ((R1+R2) .LE. D12) GO TO 1000
 IF ((R2+R3) .LE. D23) GO TO 1000
 IF ((R3+R4) .LE. D34) GO TO 1000
 IF ((R4+R1) .LE. D41) GO TO 1000
 CLA1=ALOG((R1+R2-D12)/(R1+R2+D12))
 CLA2=ALOG((R2+R3-D23)/(R2+R3+D23))
 CLA3=ALOG((R3+R4-D34)/(R3+R4+D34))
 CLA4=ALOG((R4+R1-D41)/(R4+R1+D41))
 TUZ=CY12*CLA1+CY23*CLA2+CY34*CLA3+CY41*CLA4
 TUY=CX12*CLA1+CX23*CLA2+CX34*CLA3+CX41*CLA4
 TUZ=0.
 IF(ABS(Z/ST)-.010) 375,361,361
 361 ZSQ=Z**2
 E1=ZSQ+(X-X1)**2
 E2=ZSQ+(X-X2)**2
 E3=ZSQ+(X-X3)**2
 E4=ZSQ+(X-X4)**2
 H1=(Y-Y1)*(X-X1)
 H2=(Y-Y2)*(X-X2)
 H3=(Y-Y3)*(X-X3)
 H4=(Y-Y4)*(X-X4)
 IF(C112)363,363,364
 363 WS1=(CM12*E1-H1)/(Z*R1)
 WS2=(CM12*E2-H2)/(Z*R2)
 AT1=ATAN(WS1)
 AT2=ATAN(WS2)
 TUZ=AT1-AT2
 364 IF(C123)365,365,367
 365 AT3=ATAN((CM23*E2-H2)/(Z*R2))
 AT4=ATAN((CM23*E3-H3)/(Z*R3))
 TUZ=TUZ+AT3-AT4
 367 IF(C134)366,366,369
 368 AT5=ATAN((CM34*E3-H3)/(Z*R3))
 AT6=ATAN((CM34*E4-H4)/(Z*R4))
 TUZ=TUZ+AT5-AT6
 369 IF(C141)370,370,375
 370 AT7=ATAN((CM41*E4-H4)/(Z*R4))
 AT8=ATAN((CM41*E1-H1)/(Z*R1))
 TUZ=TUZ+AT7-AT8
 375 GO TO 450
 C G. COMPUTE VELOCITY COEF. BY QUADRAPOLE METHOD

```

400 RPQ3=RPQ**3
RPQ7=(RPQ3**2)*RPQ
WS1= X/RPQ3
XSQ=X**2
YSQ=Y**2
ZSQ=Z**2
PS=YSQ+ZSQ-4.*XSQ
QS=XSQ+ZSQ-4.*YSQ
WS2=X*(9.*PS+30.*XSQ)/RPQ7
WS3=3.*Y*PS/RPQ7
WS4=3.*X*QS/RPQ7
TUX=A*WS1-C1XY*WS3-C1XX*WS2-C1YY*WS4
WS1=Y/RPQ3
WS2=Y*(9.*QS+30.*YSQ)/RPQ7
TUY=A*WS1-C1XX*WS3-C1XY*WS4-C1YY*WS2
TUZ=Z*(R/RPQ3-3.*C1XX*PS-5.*C1XY*X*Y+C1YY*QS)/RPQ7
450 UX(I$)=TUX*XX+TUY*XY+TUZ *XN
UY(I$)=TUX*YX+TUY*YY+TUZ*YN
UZ(I$)=TUX*ZX+TUY*ZY+TUZ*ZN
GO TO 470
C      H COMPUTE VELOCITY COEF. BY MONPOLE METHOD
460 ARPQ3=A/(RPQ**3)
UX(I$)=(XCO-XC)*ARPQ3
UY(I$)=(YCO-YC)*ARPQ3
UZ(I$)=(ZCO-ZC)*ARPQ3
C      I. REFLECT NULL PT. IN PLANE OF SYMETRY
470 GO TO(480,485,490,495,500,505,510,515),IS
C      DO LOOPS SET UP TO FORCE USE OF INDEX REGISTERS
480 J1=JU
J2=JC
UDY=UX(1)
UDY=UY(1)
UDZ=UZ(1)
U1(J1)=UX(1)
U2(J1)=UX(1)
U3(J1)=UX(1)
U1(J1+1)=UY(1)
U2(J1+1)=UY(1)
U3(J1+1)=UY(1)
U1(J1+2)=UZ(1)
U2(J1+2)=UZ(1)
U3(J1+2)=UZ(1)
IF(SYM) 530,530,481
481 IS=2
C      XZ SYMETRY
YCO=-YCO
GO TO 345
495 IF(SYM-1)517,517,485
C      XY SYMETRY
486 IS=3
ZCO=-ZCO
GO TO 345
490 IS=4
YCO=-YCO
GO TO 345
495 IF(SYM-2)516,516,495
C      YZ SYMETRY
496 IS=5
XCO=-XCO
GO TO 345
500 IS=6
YCO=-YCO

```

```

      GO TO 345
505 IS=7
      ZCQ=-ZCQ
      GO TO 345
510 IS=8
      YCQ=-YCQ
      GO TO 345
C      J. ADD CONTRIBUTIONS OF ALL REFLECTIONS
515 U1(J1)=U1(J1)+UX(8)+UX(7)+UX(6)+UX(5)
      U2(J1)=U2(J1)-UX(8)+UX(7)+UX(6)-UX(5)
      U3(J1)=U3(J1)-UX(8)-UX(7)+UX(6)+UX(5)
      U1(J1+1)=U1(J1+1)-UY(8)+UY(7)+UY(6)-UY(5)
      U2(J1+1)=U2(J1+1)+UY(8)+UY(7)+UY(6)+UY(5)
      U3(J1+1)=U3(J1+1)+UY(8)-UY(7)+UY(6)-UY(5)
      U1(J1+2)=U1(J1+2)-UZ(8)-UZ(7)+UZ(6)+UZ(5)
      U2(J1+2)=U2(J1+2)+UZ(8)-UZ(7)+UZ(6)-UZ(5)
      U3(J1+2)=U3(J1+2)+UZ(8)+UZ(7)+UZ(6)+UZ(5)
516 U1(J1)=U1(J1)+UX(4)+UX(3)
      U2(J1)=U2(J1)+UX(4)-UX(3)
      U3(J1)=U3(J1)-UX(4)-UX(3)
      U1(J1+1)=U1(J1+1)+UY(4)-UY(3)
      U2(J1+1)=U2(J1+1)+UY(4)+UY(3)
      U3(J1+1)=U3(J1+1)-UY(4)+UY(3)
      U1(J1+2)=U1(J1+2)-UZ(4)-UZ(3)
      U2(J1+2)=U2(J1+2)-UZ(4)+UZ(3)
      U3(J1+2)=U3(J1+2)+UZ(4)+UZ(3)
517 U1(J1)=U1(J1)+UX(2)
      U2(J1)=U2(J1)-UX(2)
      U3(J1)=U3(J1)+UX(2)
      U1(J1+1)=U1(J1+1)-UY(2)
      U2(J1+1)=U2(J1+1)+UY(2)
      U3(J1+1)=U3(J1+1)-UY(2)
      U1(J1+2)=U1(J1+2)+UZ(2)
      U2(J1+2)=U2(J1+2)-UZ(2)
      U3(J1+2)=U3(J1+2)+UZ(2)
530 C1(J2)=XNQ*U1(J1)+YNQ*U1(J1+1)+ZNQ*U1(J1+2)
      C2(J2)=XNQ*U2(J1)+YNQ*U2(J1+1)+ZNQ*U2(J1+2)
      C3(J2)=XNQ*U3(J1)+YNQ*U3(J1+1)+ZNQ*U3(J1+2)
540 JV=JV+3
541 JC=JC+1
C      D. WRITE COEFFICIENTS
C      E. WRITE COEF. ON TAPE OR DRUM IF STORAGE AREA IS FULL
545 IF(JV-1001)570,555,555
555 JV=2
      U1(1)=BLK
      U2(1)=BLK
      U3(1)=BLK
      IF(BLK=636.0) 560,563,566
560 WRITE(01) BLK,U1,U2,U3
      GO TO 562
563 REWIND 01
566 WRITE(11) BLK,U1,U2,U3
568 BLK=BLK+1
570 IF(JC=901)580,571,571
571 IDW=IDW+1
      WRITE(02) IDW,C1
      WRITE(03) IDW,C2
      WRITE(09) IDW,C3
576 JC=1
580 KO=KO+7
      L=L+1
C      L END OF LOOP OVER NULL PTS.

```

```

      IF(KQ-KM) >343,343,581
581 C1(JC)=0
      C2(JC)=0
      C3(JC)=0
      IF(KQ-KMM)>541,585,585
585 P=P+1
      K=K+7
      J=J+20
      IF(K-KM) >536,586,600
C          M. END OF LOOP OVER QUADS.
C          M. READ NEXT BLOCK OF B ARRAY IF NEEDED
586 IF(J=241)296,590,590
590 READ(04)(B(I), I=1,241)
      J=2
      IF(B(1)=P)595,296,595
595 WRITE (6,98) B(1),P
      STOP
600 IF(BLK=636.0) 610,620,630
C          O. WRITE REMAINING COEF. ON TAPE OR DRUM
610 WRITE(01)BLK,U1,U2,U3
      REWIND 01
      GO TO 640
620 REWIND 01
630 WRITE(11) BLK,U1,U2,U3
      REWIND 11
640 WRITE (02) IDW,C1
      WRITE (03) IDW,C2
      WRITE (09) IDW,C3
      REWIND 02
      REWIND 03
      REWIND 04
      REWIND 08
      REWIND 09
C          P. TRANSFER TO PFPS3
      GO TO 5000
1000 WRITE(6,2000) L,P
2000 FORMAT(3H L= ,15,20X,3H P= ,F5.1)
5000 CONTINUE
      STOP 2
      END
      FUNCTION SQ2F(X1,X2,Y1,Y2,Z1,Z2)
      X=X1-X2
      Y=Y1-Y2
      Z=Z1-Z2
      RS=Z**2+Y**2+X**2
      R=ABS(X)+ABS(Y)+ABS(Z)+ 1.0E-20
      R=R+RS/R
      R=.25*R+RS/R
      R= R+RS/R
      SQ2F=.25*R+RS/R
      RETURN
      END

```

APPENDIX III - XYZPF SECTION PF3

```
PROGRAM PFP3(OUTPUT=128,TAPE02,TAPE08,TAPE09,
1           TAPE12,TAPE03,TAPE6=OUTPUT,TAPE2=TAPE02,
2           TAPE5=TAPE08,TAPE9=TAPE09,TAPE3=TAPE03)
C
C XYZ POTENTIAL FLOW PROGRAM VERSION 1 SECTION 3
C SOLVES MATRIX EQUATION FOR SOURCE DENSITY
C
COMMON SN(654),VIP(650),S(5,650),PROB(15),WS(220),DM(650),
1           B(220),COEF(900),XN(650),YN(650),ZN(650)
EQUIVALENCE (WS(213),EPS), (KK,B(201))
EQUIVALENCE (M1X,WS(205)),(M1Y,WS(206)),(M1Z,WS(207))
EQUIVALENCE (WS(201),NP),(WS(208),IPS),(WS(212),IPF),(WS(211),KM),
1(KM,MK)
5 FORMAT(49HXYZ POTENTIAL FLOW PROGRAM SECTION 3, VERSION 4 )
WRITE (6,5)
READ  (03)(PROB(I),I=1,15)
WRITE  (6,1001)(PROB(I),I=1,15)
1001 FORMAT(1HO,15A4)
READ (03) (WS(I),I=1,220), IEDIT3, IEDIT4
READ  (03)(SKIP,SKIP,SKIP,XN(I),YN(I),ZN(I),SKIP,I=1,NP)
D=-.5/3.14159265
READ  (03)(DM(I),I=1,NP)
K1=1
K2=NP
240 FORMAT (18H0CHANGES IN PROB -, 15A4)
IF(IPS)1220,1220,1231
1231 READ  (12)( B(K),K=1,15)
WRITE  (6,240)(B(K),K=1,15)
READ  (12)( B(K),K=1,220)
READ  (12)SKIP
READ  (12)SKIP
K1=IPS
K2=IPF
C     A. SET CONDITIONS FOR FLOW OF -1 IN X DIRECTION
1220 FX=-1
FY=0
FZ=0
NF=-1
C     B. COMPUTE INITIAL APPROXIMATION TO THE SOURCE
1240 DO 1250 K=1,NP
VIP(K)=XN(K)*FX+YN(K)*FY+ZN(K)*FZ-DM(K)
S(5,K)=-VIP(K)*.11936
C     C. SET PARTIAL SUM VECTOR TO ZERO
1250 SN(K)=0.
SN(NP+1)=0.
SN(NP+2)=0.
SN(NP+3)=0.
SN(NP+4)=0.
WRITE(6,997) FX,FY,FZ
WRITE (6,998)
998 FORMAT(27H0ITERATION    SUM OF CHANGES ,9X,1HR,10X,2HE1,10X,2HE2)
IT=1
IC=5
IF (IPS) 1260,1260,1255
1255 READ(12) (S(5,K), K=1,KK)
DO 1256 K=1,KK
DO 1256 I=1,4
1256 S(I,K)=S(5,K)
C     D. START ITERATION
1260 BAND=0
```

```

      IF (NF) 1261, 1262, 1263
1261 READ (02)IDW,COEF
      GO TO 1264
1262 READ (08)IDW,COEF
      GO TO 1264
1263 READ (09)IDW,COEF
1264 J=0
C      D.  READ FIRST BLOCK OF COEF
C      E.  START LOOP OVER QUADS.
      DO 1290 K=1,NP
C      F.  PICK UP SOURCE DENSITY
      SP=S(IC,K)
C      G.  START LOOP OVER NULL PTS.
      DO 1290 KP=1,NP,5
      IF(J=900)80,65,65
      65 IF (NF)67,68,69
      67 READ (02)IDW,COEF
      GO TO 70
      68 READ (08)IDW,COEF
      GO TO 70
      69 READ (09)IDW,COEF
      70 J=0
C      H.  COMPUTE PARTIAL SUM FOR NEXT 5 PTS.
      80 SN(KP)=SN(KP)+COEF(J+1)*SP
      SN(KP+1)=SN(KP+1)+COEF(J+2)*SP
      SN(KP+2)=SN(KP+2)+COEF(J+3)*SP
      SN(KP+3)=SN(KP+3)+COEF(J+4)*SP
      SN(KP+4)=SN(KP+4)+COEF(J+5)*SP
      J=J+5
C      J.  END OF LOOP OVER NULL PTS.
C      K.  END OF LOOP OVER QUADS.
1290 CONTINUE
C      L.  COMPUTE NEW SOURCE
      IF (NF) 91,92,93
      91 REWIND 02
      GO TO 94
      92 REWIND 08
      GO TO 94
      93 REWIND 09
      94 PASS=1.0
      SUM=0.
      DO 100 K=K1,K2
      SN(K)=( SN(K)+VTP(K) )*0
      TEST=ABS(SN(K)-S(IC,K))
      SUM=SUM+TEST
      IF (TEST .GT. EPS) PASS=-1.0
100 CONTINUE
      IF (PASS .EQ. 1.0) GO TO 180
      IF (IT.GE.MIX) GO TO 180
      IF (IEDIT3 .EQ. 0) WRITE(6,99) IT,SUM
      IT=IT+1
      IC=IC-1
      IF (IC .EQ. 0) GO TO 120
      DO 110 K=K1,K2
      S(IC,K)=SN(K)
110 SN(K)=0.
      GO TO 1260
120 A=0.
      B1=0.
      B2=0.
      DA=0.
      D1=0.

```

```

D2=0.
DO 140 K= K1,K2
DS9=2*S(1,K)-SN(K)-S(2,K)
IF(DS9 .GT. 0.) GO TO 122
R=A+S(2,K)-S(1,K)
DA=DA-DS9
GO TO 125
122 R =A +S(1,K)-S(2,K)
DR=DA+DS9
125 DS1=S(4,K)-S(3,K)
DS2=S(3,K)-S(2,K)
DS3=DS1-DS2
DS5=S(2,K)-S(1,K)
DS5=DS2-DS5
DS6=DS1-DS5
DS4=DS2-S(1,K)+SN(K)
DS7=DS3*DS4-DS5*DS6
DS8=DS6*DS5-DS4*DS3
IF(DS7 .GT. 0.) GO TO 128
B1=B1-DS1*DS4+DS2*DS6
D1=D1+DS7
GO TO 130
128 B1=B1+DS1*DS4-DS2*DS6
D1=D1+DS7
130 IF (DS8 .GT. 0.) GO TO 132
B2=B2-DS1*DS5+DS2*DS3
D2=D2-DS8
GO TO 140
132 B2=B2+DS1*DS5-DS2*DS3
D2=D2+DS8
140 CONTINUE
R=A/DA
B1=B1/D1
B2=B2/D2
IF (IT .EQ. 6) GO TO 155
RA=R-AS
RR=ABS(RA)
IF (RA .GT. .02) GO TO 148
DO 145 K=K1,K2
S(5,K)=R*(SN(K)-S(1,K))+S(1,K)
145 SN(K)=0.
WRITE(6,6000)
6000 FORMAT(29X,17HA EXTRAPOLATION    )
GO TO 160
148 BB1=B1-BB1
BB1=ABS(BB1)
BB1=50.*BB1
BB2=B2-BB2
BB2=ABS(BB2)
BB2=50.*BB2
BBB=ABS(BB1) + ABS(BB2)
IF (< (BB1 .GT. BBB) .OR. (BB2 .GT. BBB) ) GO TO 155
DO 150 K=K1,K2
S(5,K)=S(2,K)+B1*(S(1,K)-S(2,K))+B2*(SN(K)-S(2,K))
150 SN(K)=0.
WRITE(6,7000)
7000 FORMAT(29X,17HB EXTRAPOLATION    )
GO TO 160
155 DO 156 K=K1,K2
S(5,K)=SN(K)
158 SN(K)=0.
160 IC=5

```

```
      WRITE(6,161) R, B1, B2
161 FORMAT(29X,3E12.3)
      AS=R
      BS1=B1
      BS2=B2
      GO TO 1260
180 WRITE(6,99) IT,SUM
      DO 182 K=K1,K2
182 S(1,K)=SN(K)
      WRITE(03) (S(1,K), K=1,NP)
99 FORMAT(4X,13,E18.5)
997 FORMAT (13H0 X VELOCITY=,F4.1,15H      Y VELOCITY=,F4.1,
           115H      Z VELOCITY=,F4.1)
      IF(FZ>1400,1390,1400
C      P1  IF THIS WAS NOT LAST FLOW, SET FOR NEXT FLOW
1390 FZ=FY
      FY=FX
      FX=0
      MX=M1Y
      MY=M1Z
      NF=NF+1
      GO TO 1240
1400 REWIND 03
C      P2  READ IN PFFS4 AND TRANSFER TO IT
      STOP 3
      END
```

APPENDIX I U - XYZPF SECTION PF4

```
PROGRAM PFP4(OUTPUT,TAPE6=OUTPUT,TAPE03,TAPE01,TAPE11,
1           TAPE3=TAPE03,TAPE1=TAPE01)

C XYZ POTENTIAL FLOW PROGRAM VERSION 1 SECTION 4
C COMPUTES VELOCITIES AND PRESSURE COEFFICIENTS FOR
C POINTS ON THE BODY

C COMMON      UX1(650),VY1(650),VZ1(650),    UX2(650),VY2(650),VZ2(650)
1           ,UX3(650),VY3(650),VZ3(650),    S1(650), S2(650), S3(650)
2           , X(650),   Y(650),   Z(650),     T4(650), T5(650), T6(650)
3           , DM(650)
DIMENSION PROB(15),WS(220),CV1(1000),CV2(1000),CV3(1000)
EQUIVALENCE (WS(201),NP),(WS(211),KM),(WS(217),UX1),(WS(218),VY1),
1(WS(219),VZ1),(WS(208),IPS),(WS(212),IPF)
EQUIVALENCE (M1X,WS(205)),(M1Y,WS(206)),(M1Z,WS(207))
5 FORMAT(49HXYZ POTENTIAL FLOW PROGRAM SECTION 4, VERSION 4 )
WRITE (6,5)
READ  (03)(PROB(I),I=1,15)
100 FORMAT(1H ,15A4)
WRITE      ( 6,100)(PROB(I),I=1,15)
C A. READ PARAMETERS AND SOURCE
READ (03)(WS(I),I=1,220),IEDIT3,IEDIT4
READ  (03)(X(I),Y(I),Z(I),T4(I),T5(I),T6(I),SKIP, I=1,NP)
READ  (03)(DM(I),I=1,NP)
D=-.5/3.14159265
READ  (03)(S1(I),I=1,NP)
READ  (03)(S2(I),I=1,NP)
READ (03)(S3(I),I=1,NP)
J=1
K1=1
K2=NP
IF(IPS>108,108,102
102 K1=IPS
K2=IPF
103 BBR=1.
C B. READ FIRST BLOCK OF COEF.
1TAPE=01
READ (01) BB,CV1,CV2,CV3
IF (BB-BBR)>300,120,300
120 DO 125I=K1,K2
UX1(I)=-1.0      -S1(I)*T4(I) /D
UY1(I)=          -S1(I)*T5(I) /D
UZ1(I)=          -S1(I)*T6(I) /D
UX2(I)=          -S2(I)*T4(I) /D
UY2(I)=-1.0      -S2(I)*T5(I) /D
UZ2(I)=          -S2(I)*T6(I) /D
UX3(I)=          -S3(I)*T4(I) /D
UY3(I)=          -S3(I)*T5(I) /D
125 UZ3(I)=-1.0  -S3(I)*T6(I) /D
C C. SET UP LOOP OVER QUADS.
JC=2
C D. PICK UP SOURCE
130 S1J=S1(J)
S2J=S2(J)
S3J=S3(J)
C E. SET UP LOOP OVER NULL PTS.
DO 180 JP=K1,K2
C F. COMPUTE PARTIAL SUM FOR 3 COMPONENTS OF 3 VELOCITIES
UX1(JP)=UX1(JP)+S1J*CV1(JC)
UY1(JP)=UY1(JP)+S1J*CV1(JC+1)
```

```

UZ1(JP)=UZ1(JP)+S1J*CU1(JC+2)
UX2(JP)=UX2(JP)+S2J*CU2(JC)
UY2(JP)=UY2(JP)+S2J*CU2(JC+1)
UZ2(JP)=UZ2(JP)+S2J*CU2(JC+2)
UX3(JP)=UX3(JP)+S3J*CU3(JC)
UY3(JP)=UY3(JP)+S3J*CU3(JC+1)
UZ3(JP)=UZ3(JP)+S3J*CU3(JC+2)
JC=JC+3
C      G. READ MORE COEF. IF NEEDED.
      IF (JC-1000)>180, 135, 135
135 JC=2
      IF(BBR-635.0) 140, 150, 155
140 READ (01) BB,CU1,CU2,CU3
      GO TO 160
150 REWIND 01
155 READ (11) BB,CU1,CU2,CU3
160 BBR=BBR+1.
      IF (BBR-BB) 300, 180, 300
C      H. END OF LOOP OVER NULL PTS.
180 CONTINUE
      J=J+1
C      I. END OF LOOP OVER QUADS.
      IF(J-NP)>130, 130, 200
200 IF(BBR-635.0) 231,231,232
231 REWIND 01
      GO TO 233
232 REWIND 11
C      K. EDIT THE VELOCITIES ETC. AND WRITE THEM ON TAPE
233 WRITE (03)(UX1(1),UY1(1),UZ1(1)) ,I=1,NP)
      WRITE (03)(UX2(1),UY2(1),UZ2(1)) ,I=1,NP)
      WRITE (03)(UX3(1),UY3(1),UZ3(1)) ,I=1,NP)
235 FORMAT(1H1, 15A4, 8H PAGE =,115)
      REWIND 03
      IP=K1+49
      IS=K1
      IPAGE=1
      IF (IEDIT4 .EQ. 1) GO TO 293
      IF (MIX .LE. 0 ) GO TO 265
242 FORMAT(8H0 X FLOW)
240 FORMAT(4H PT., 10X, 1HX, 9X, 1HY, 9X, 1HZ, 1BX, 2HUX, 8X, 2HUY, 8X, 2HUZ, 10X,
1 5HABS.U, 7X, 2HCP, 6X, 6HSOURCE, 4X, 8HU NORMAL)
245 FORMAT (1X,13,4X,3F10.5,4X,3F10.5,1F13.5,2F11.5,E12.2)
250 IF(IP-K2)>255, 255, 260
C      J. COMPUTE PRESSURE AND ABS. VALUE OF VELOCITY
255 WRITE (6,235)(PROB(I),I=1,15),IPAGE
      WRITE (6,242)
      WRITE (6,240)
      DO 257 I=IS,IP
      USQ=UX1(1)**2+UY1(1)**2+UZ1(1)**2
      UM=(ABS(UX1(1))+ABS(UY1(1))+ABS(UZ1(1)))*.79
      UM=UM+USQ/UM
      VM=.25*UM+USQ/UM
      VM=.5*(VM+USQ/VM)
      CP=1.-USQ
      UNR=UX1(1)*T4(1) +UY1(1)*T5(1) +UZ1(1)*T6(1)
257 WRITE (6,245) 1,X(1),Y(1),Z(1),UX1(1),UY1(1),UZ1(1),UM
      1 ,CP ,S1(1),UNR
      IS=IS+50
      IP=IP+50
      IPAGE=IPAGE+1
      IF(K2-IS) 265,260,250
260 IP=K2

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```

      GO TO 255
265 IP=K1+49
      IS=K1
      IF (M1Y .LE. 0 ) GO TO 280
267 IF(IP-K2)275,275,270
270 IP=K2
275 WRITE          (6,235)(PROB(1),I=1,15),IPAGE
      WRITE          (6,277)
277 FORMAT(8HO Y FLOW)
      WRITE          (6,240)
      DO 278 I=IS,IP
      USQ=UX2(I)**2+UY2(I)**2    +UZ2(I)**2
      UM=(ABS(UX2(I))+ABS(UY2(I))+ABS(UZ2(I)))*.79
      UM=UM+USQ/UM
      VM=.25*UM+USQ/UM
      VM=.5*(VM+USQ/UM)
      UNR =UX2(I)*T4(I)    +UY2(I)*T5(I)    +UZ2(I)*T6(I)
      CP =1.-USQ
278 WRITE (6,245) I,X(I),Y(I),Z(I),UX2(I),UY2(I),UZ2(I),UM
      1 ,CP ,S2(I),UNR
      IS=IS+50
      IP=IP+50
      IPAGE=IPAGE+1
      IF(IS-K2)267,267,280
280 IP=K1+49
      IS=K1
      IF (M1Z .LE. 0 ) GO TO 293
282 IF(IP-K2)290,290,285
285 IP=K2
290 WRITE          (6,235)(PROB(1),I=1,15),IPAGE
      WRITE          (6,292)
292 FORMAT(8HO Z FLOW)
      WRITE          (6,240)
      DO 291 I=IS,IP
      USQ=UX3(I)**2+UY3(I)**2+UZ3(I)**2
      UM=(ABS(UX3(I))+ABS(UY3(I))+ABS(UZ3(I)))*.79
      UM=UM+USQ/UM
      VM=.25*UM+USQ/UM
      VM=.5*(VM+USQ/UM)
      CP =1.-USQ
      UNR =UX3(I)*T4(I)    +UY3(I)*T5(I)    +UZ3(I)*T6(I)
291 WRITE (6,245) I,X(I),Y(I),Z(I),UX3(I),UY3(I),UZ3(I),UM
      1 ,CP ,S3(I),UNR
      IS=IS+50
      IP=IP+50
      IPAGE=IPAGE+1
      IF(IS-K2)282,282,293
C     L. CHECK FOR A FOURTH FLOW
293 J = 1
294 UX1 = WS(J)
      UY1 = WS(J+20)
      UZ1 = WS(J+40)
295 IF ((UX1)**2+UY1**2+UZ1**2) .GT. 400,400,301
301 IS=K1
      IP=K1+49
320 IF (IP-K2)330,325,325
C     M. EDIT THE VELOCITY AND PRESSURE FOR FOURTH FLOW
325 IP=K2
330 WRITE          (6,235)(PROB(1),I=1,15),IPAGE
      WRITE          (6,315)UX1,UY1,UZ1
      WRITE          (6,340)
C     N. COMPUTE VELOCITY AND PRESSURE FOR FOURTH FLOW

```

```

DO 333 I=IS, IP
  UX4 =- (UX1*UX1(I)+UY1*UX2(I)+UZ1*UX3(I))
  UY4 =- (UX1*UY1(I)+UY1*UY2(I)+UZ1*UY3(I))
  UZ4 =- (UX1*UZ1(I)+UY1*UZ2(I)+UZ1*UZ3(I))
  USQ=UX4 **2+UY4 **2+UZ4 **2
  UM=(ABS(UX4 )+ABS(UY4 )+ABS(UZ4 ))*.79
  VM=UM+USQ/UM
  VM=.25*VM+USQ/VM
  UM = .5*(UM+USQ/UM)
  CP = 1.-<USQ>/(<UY1**2+UZ1**2+UX1**2>
333 WRITE (6,345) I,X(I),Y(I),Z(I),UX4 ,UY4 ,UZ4 ,VM
  1 ,CP
  IS=IS+50
  IP=IP+50
  IPAGE=IPAGE+1
  IF(K2-IS)>350,325,320
325 J = J+1
  GO TO 294
315 FORMAT(19HG ONSET FLOW, UX1=F6.3,2X,4HUY1=F6.3,2X,4HUZ1=F6.3)
340 FORMAT(4H PT., 10X,1HX,9X,1HY,9X,1HZ,13X,2HUX,8X,2HUY,8X,2HUZ,10X,
  1.5HAB3.U, 7X,2HCP)
345 FORMAT (1X,13,4X,3F10.5,4X,3F10.5,1F13.5,1F11.5)
400 CONTINUE
  GO TO 5000
500 CONTINUE
  WRITE (6,310) 1TAPE,BBR,BB
310 FORMAT (6H1TAPE ,12,17H OUT OF POSITION/114,6F6.1)
5000 CONTINUE
  STOP 4
  END

```

APPENDIX U - XYZPF SECTION PFS

```
PROGRAM PFPS(INPUT=128,OUTPUT=128,TAPE03,TAPE04,
1TAPE5=INPUT,TAPE6=OUTPUT,TAPE3=TAPE03,TAPE4=TAPE04)
C
C XYZ POTENTIAL FLOW PROGRAM VERSION 4 SECTION 5
C COMPUTES VELOCITIES AND PRESSURE COEFFICIENTS FOR
C OFF BODY POINTS
C
COMMON B(241),      XP(500),YP(500),ZP(500),UX1(500),WS(220)
1,VY1(500),VZ1(500),UX2(500),VY2(500),VZ2(500),UX3(500),VY3(500)
2,VZ3(500),UX(8),VY(8),VZ(8),S1(650),S2(650),S3(650),U1(3),U2(3),U3
3(3),PROB(15),XN(650),YN(650),ZN(650),TX(650),TY(650)
4,TZ(650)
EQUIVALENCE (KM,WS(211)),(KM,MK),(NP,WS(201)),(SYM,WS(210))
1,(V2,Y3),(M1X,WS(205)),(M1Y,WS(206)),(M1Z,WS(207))
INTEGER PAGE
INTEGER SYM
5 FORMAT(49HXYZ POTENTIAL FLOW PROGRAM SECTION 5, VERSION 4 )
WRITE(6,5)
C      A. READ INPUT
READ(5,25) NOBP,IEDITS,IREAD
C      A. READ THE OFF BODY POINTS
NOB=NOBP
DO 10 I=1,NOB
READ(5,26) XP(I),YP(I),ZP(I)
IF (EOF(5).EQ.0.) GO TO 10
NOBP=I-1
WRITE(6,9) NOBP,NOB
9 FORMAT(1H0,15,31H OFF BODY POINTS SPECIFIED NOT ,13)
GO TO 11
10 CONTINUE
11 CONTINUE
25 FORMAT(314)
26 FORMAT(3F12.5)
P=1.
READ (03) (PROB(I),I=1,15)
WRITE (6,90) (PROB(I),I=1,15)
90 FORMAT(1H0,18A4)
WRITE(6,91) NOBP,IEDITS,IREAD
91 FORMAT(8HONOBP =,14 /8H IEDIT5=,14/8H IREAD =,14)
WRITE(6,92)
92 FORMAT(17H0 OFF BODY POINTS / 4H PT.,11X,1HX,12X,1HY,12X,1HZ)
WRITE(6,93) (1,XP(I),YP(I),ZP(I),I=1,NOBP)
93 FORMAT(1X,13,2X,3F13.5)
C      B. READ THE PARAMETERS, T ARRAY AND SOURCE FROM TAPE 31
READ (03) (WS(I),I=1,220)
READ (03) (TX(I),TY(I),TZ(I),XN(I),YN(I),ZN(I),TC(I),I=1,NP)
READ (03) SKIP
C FORMERLY: WS(220).EQ. 2.
C
IF(WS(220).EQ. 5.) READ(03) SKIP
READ (03) (S1(I),I=1,NP)
READ (03) (S2(I),I=1,NP)
READ (03) (S3(I),I=1,NP)
C      C. READ THE FIRST BLOCK OF THE B ARRAY
READ (04) (B(I),I=1,241)
K=1
J=1
DO 100 I=1,NOBP
C      D. SET THE PARTIAL VELOCITY TO THE FREE STREAM VELOCITY
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```

UX1(1)=-1.0
VY1(1)=0.
UZ1(1)=0.
UX2(1)=0.
VY2(1)=-1.0
UX3(1)=0.0
VY3(1)=0.0
UZ3(1)=-1.0
100 UZ2(1)=0.
290 IF(B(1)-P>291,295,291
291 WRITE(6,98) B(1),P
98 FORMAT(28H0 POINTS OUT OF ORDER B(1)=,1F4.0,4H P=,1F4.0)
STOP
C      E. START LOOP OVER THE QUADS.
295 J=2
C      F1 PICK UP QUAD. INFORMATION
296 X1=B(J)
Y1=B(J+1)
X2=B(J+2)
Y2=B(J+3)
X3=B(J+4)
Y3=B(J+5)
X4=B(J+6)
Y4=B(J+7)
XC=TX(K)
YC=TY(K)
ZC=TZ(K)
R =TR(K)
XX=B(J+8)
YX=B(J+9)
ZX=B(J+10)
XY=B(J+11)
YY=B(J+12)
ZY=B(J+13)
C      F2 COMPUTE LENGTH OF SIDES OF QUAD.
D12=SQ2F(X1,X2,Y1,Y2,0.,0.)
D23=SQ2F(X2,X3,Y2,Y3,0.,0.)
D34=SQ2F(X3,X4,Y3,Y4,0.,0.)
D41=SQ2F(X4,X1,Y4,Y1,0.,0.)
C      F3 COMPUTE SLOPE OF SIDES
IF(X2-X3)305,300,305
300 C123=1.
GO TO 310
305 CM23=(Y2-Y3)/(X2-X3)
C123=0.
310 IF(X3-X4)315,311,315
311 C134=1.
GO TO 320
315 CM34=(Y4-Y3)/(X4-X3)
C134=0.
320 IF(X4-X1)325,321,325
321 C141=1.
GO TO 330
325 CM41=(Y1-Y4)/(X1-X4)
C141=0.
330 IF(X1-X2)335,331,335
331 C112=1.
GO TO 340
335 CM12=(Y2-Y1)/(X2-X1)
C112=0.
C      F4 COMPUTE QUADRPOLE MOMENTS
340 C1XX=B(J+17)
C1XY=B(J+18)

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C      C1YY=B(J+1Q)
      F5  COMPUTE SIN AND COS OF SLOPE ANGLE FOR EACH SIDE
      CY12=(Y2-Y1)/D12
      CY23=(Y3-Y2)/D23
      CY34=(Y4-Y3)/D34
      CY41=(Y1-Y4)/D41
      CX12=(X1-X2)/D12
      CX23=(X2-X3)/D23
      CX34=(X3-X4)/D34
      CX41=(X4-X1)/D41
C      F6  COMPUTE MAX DIAGONAL
      ST=SQ2F(X1,X3,Y1,Y3,0.,0.)
      ST2=SQ2F(X2,X4,Y2,Y4,0.,0.)
      IF(ST-ST2)G41,342,342
      341 ST=ST2
C      G.  START LOOP OVER THE OFF BODY POINTS
      342 DO 530 JQ=1,NDBP
         IS=1
         XCQ=XP(JQ)
         YCQ=YF(JQ)
         ZCQ=ZP(JQ)
         J1=1
         345 RPO=SQ2F(XC,XCQ,YC,YCQ,ZC,ZCQ)
C      H.  DETERMIN METHOD
         IF(RPO-ST*4)350,350,460
         350 X=(XCQ-XC)*XX+(YCQ-YC)*YY+(ZCQ-ZC)*ZX
         Y=(XCQ-XC)*XY+(YCQ-YC)*YZ+(ZCQ-ZC)*ZY
         Z=(XCQ-XC)*XN(K)+(YCQ-YC)*YN(K)+(ZCQ-ZC)*ZN(K)
         IF(RPO-ST*2.5)355,355,400
C      I.  COMPUTE INDUSED VELOCITY BY EXACT METHOD
         355 R1=SQ2F(X,X1,Y,Y1,Z,0.)
         R2=SQ2F(X,X2,Y,Y2,Z,0.)
         R3=SQ2F(X,X3,Y,Y3,Z,0.)
         R4=SQ2F(X,X4,Y,Y4,Z,0.)
         IF((R1+R2).LE.D12) GO TO 1000
         IF((R3+R2).LE.D23) GO TO 1000
         IF((R3+R4).LE.D34) GO TO 1000
         IF((R1+R4).LE.D41) GO TO 1000
         CLR1= ALOG((R1+R2-D12)/(R1+R2+D12))
         CLR2= ALOG((R2+R3-D23)/(R2+R3+D23))
         CLR3= ALOG((R3+R4-D34)/(R3+R4+D34))
         CLR4= PL0G((R4+R1-D41)/(R4+R1+D41))
         TUX=CY12*CLR1+CY23*CLR2+CY34*CLR3+CY41*CLR4
         TUY=CX12*CLR1+CX23*CLR2+CX34*CLR3+CX41*CLR4
         TUZ=0.
         IF(ABS(Z)-.001*ST)375,361,351
      361 ZS0=Z**2
         E1=ZS0+(X-X1)**2
         E2=ZS0+(X-X2)**2
         E3=ZS0+(X-X3)**2
         E4=ZS0+(X-X4)**2
         H1=(Y-Y1)*(X-X1)
         H2=(Y-Y2)*(X-X2)
         H3=(Y-Y3)*(X-X3)
         H4=(Y-Y4)*(X-X4)
         IF(C112)363,353,354
      363 WS1=(C112*E1-H1)/(Z*R1)
         WS2=(C112*E2-H2)/(Z*R2)
         AT1=ATRN(WS1)
         AT2=ATRN(WS2)
         TUZ=AT1-AT2
      364 IF(C123)366,366,367

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```

366 RT3=ATAN((CM23*E2-H2)/(Z*R2))
    RT4=ATAN((CM23*E3-H3)/(Z*R3))
    TUZ=TUZ+RT3-RT4
367 IF(C134)368,368,369
368 RT5=ATAN((CM34*E3-H3)/(Z*R3))
    RT6=ATAN((CM34*E4-H4)/(Z*R4))
    TUZ=TUZ+RT5-RT6
369 IF(C141)370,370,375
370 RT7=ATAN((CM41*E4-H4)/(Z*R4))
    RT8=ATAN((CM41*E1-H1)/(Z*R1))
    TUZ=TUZ+RT7-RT8
375 GO TO 450
C       J. COMPUTE INDUSED VELOCITY BY QUADRAPOLE METHOD
400 RPQ3=RPQ**3
    RPQ7=(RPQ3**2)*RPQ
    WS1= X/RPQ3
    XS0=X**2
    YS0=Y**2
    ZS0=Z**2
    PS=YS0+ZS0-4.*XS0
    OS=XS0+ZS0-4.*YS0
    WS2=x*(9.*PS+30.*XS0)/RPQ7
    WS3=3.*Y*PS/RPQ7
    WS4=3.*X*YS/RPQ7
    TUX=R*WS1-C1XX*WS3-C1XY*WS2-C1YY*WS4
    WS1=Y/RPQ3
    WS2=Y*(9.*OS+30.*YS0)/RPQ7
    TUV=R*WS1-C1XX*WS3-C1XY*WS4-C1YY*WS2
    TUZ=Z*(R/RPQ3-3.)*(C1XX*PS-5.*C1XY*Y+C1YY*OS)/RPQ7
450 UX(I5)=TUX*XX+TUY*XY+TUZ*XN(K)
    UY(I5)=TUX*YX+TUY*YY+TUZ*YN(K)
    UZ(I5)=TUX*ZX+TUV*ZY+TUZ*ZN(K)
    GO TO 470
C       K. COMPUTE INDUSED VELOCITY BY MONOPOLE METHOD
450 ARPQ3=R/(RPQ**3)
    UX(I5)=(XC0-XC)*ARPQ3
    UY(I5)=(YC0-YC)*ARPQ3
    UZ(I5)=(ZC0-ZC)*ARPQ3
C       L. REFLECT OFF BODY POINT IN PLANE OF SYMETRY
470 GO TO(480,485,490,495,500,505,510,515),IS
480 V1(J1)=UX(1)
    U2(J1)=UX(1)
    V3(J1)=U3(1)
    V1(J1+1)=UW(1)
    U2(J1+1)=UW(1)
    V3(J1+1)=UW(1)
    V1(J1+2)=UZ(1)
    U2(J1+2)=UZ(1)
    V3(J1+2)=UZ(1)
    IF(SYM) 525,525,481
481 IS=2
C       XZ SYMETRY
    YC0=-YC0
    GO TO 345
485 IF(SYM-1)517,517,466
C       XY SYMETRY
486 IS=3
    ZC0=-ZC0
    GO TO 345
490 IS=4
    YC0=-YC0
    GO TO 345

```

```

495 IF(SYM-2)516,516,496
C          YZ SYMETRY
496 IS=5
  XCQ=-XCQ
  GO TO 345
500 IS=6
  YCO=-YCO
  GO TO 345
505 IS=7
  ZCO=-ZCO
  GO TO 345
510 IS=8
  YCO=-YCO
  GO TO 345
C          M. ADD CONTRIBUTIONS OF ALL REFLECTIONS
515 U1(J1)=U1(J1)+UX(8)+UX(7)+UX(6)+UX(5)
  U2(J1)=U2(J1)-UX(8)+UX(7)+UX(6)-UX(5)
  U3(J1)=U3(J1)-UX(8)-UX(7)+UX(6)+UX(5)
  U1(J1+1)=U1(J1+1)-UY(8)+UY(7)+UY(6)-UY(5)
  U2(J1+1)=U2(J1+1)+UY(8)+UY(7)+UY(6)+UY(5)
  U3(J1+1)=U3(J1+1)+UY(8)-UY(7)+UY(6)-UY(5)
  U1(J1+2)=U1(J1+2)-UZ(8)-UZ(7)+UZ(6)+UZ(5)
  U2(J1+2)=U2(J1+2)+UZ(8)-UZ(7)+UZ(6)-UZ(5)
  U3(J1+2)=U3(J1+2)+UZ(8)+UZ(7)+UZ(6)+UZ(5)
516 U1(J1)=U1(J1)+UX(4)+UX(3)
  U2(J1)=U2(J1)+UX(4)-UX(3)
  U3(J1)=U3(J1)-UX(4)-UX(3)
  U1(J1+1)=U1(J1+1)+UY(4)-UY(3)
  U2(J1+1)=U2(J1+1)+UY(4)+UY(3)
  U3(J1+1)=U3(J1+1)-UY(4)+UY(3)
  U1(J1+2)=U1(J1+2)-UZ(4)-UZ(3)
  U2(J1+2)=U2(J1+2)-UZ(4)+UZ(3)
  U3(J1+2)=U3(J1+2)+UZ(4)+UZ(3)
517 U1(J1)=U1(J1)+UX(2)
  U2(J1)=U2(J1)-UX(2)
  U3(J1)=U3(J1)+UX(2)
  U1(J1+1)=U1(J1+1)-UY(2)
  U2(J1+1)=U2(J1+1)+UY(2)
  U3(J1+1)=U3(J1+1)-UY(2)
  U1(J1+2)=U1(J1+2)+UZ(2)
  U2(J1+2)=U2(J1+2)-UZ(2)
  U3(J1+2)=U3(J1+2)+UZ(2)
525 L=P
  UX1(J0)=UX1(J0)+U1(1)*S1(L)
  UY1(J0)=UY1(J0)+U1(2)*S1(L)
  UZ1(J0)=UZ1(J0)+U1(3)*S1(L)
  UX2(J0)=UX2(J0)+U2(1)*S2(L)
  UY2(J0)=UY2(J0)+U2(2)*S2(L)
  UZ2(J0)=UZ2(J0)+U2(3)*S2(L)
  UX3(J0)=UX3(J0)+U3(1)*S3(L)
  UY3(J0)=UY3(J0)+U3(2)*S3(L)
  UZ3(J0)=UZ3(J0)+U3(3)*S3(L)
530 CONTINUE
C          N. END OF LOOP OVER OFF BODY POINTS
585 P=P+1
  K=K+1
  J=J+20
  IF(K-NP .GT. 586,586,599
586 IF(J-241)296,590,597
C          O. READ NEXT BLOCK OF B ARRAY IF NEEDED
590 READ (04) (B(I)), I=1,241
  J=2

```

```

IF(B(1)=P)291,296,291
C      P. END OF LOOP OVER QUADS
599 CONTINUE
PAGE = 1
IF (<EDIT5 .EQ. 1>) GO TO 825
601 FORMAT(4H PT.,11X,1HX,12X,1HY,12X,1HZ,14X,2HUX,11X,2HUY,11X,2HUZ
1,14X,2HCP)
602 FORMAT(7H X FLOW)
603 FORMAT(7H Y FLOW)
604 FORMAT(7H Z FLOW)
605 FORMAT(1H1,15H4,10X,15H0FF BODY POINTS ,10X,5HPAGE ,13)
IF (MIX.EQ.0) GO TO 700
WRITE(6,605) PROB,PAGE
WRITE (6,602)
WRITE (6,601)
LINE=1
LAST=53
606 IF(NOBP-LAST)>607,610,610
607 LAST=NOBP
610 DO 615 I=LINE,LAST
611 FORMAT(1X,113,2X,3F13.5,2X,3F13.5,3X,F13.5)
C      Q. COMPUTE PRESSURE AND EDIT 3 BASIC FLOWS
CP1=1.- $(UX1(I)**2+UY1(I)**2+UZ1(I)**2)$ 
615 WRITE (6,611) I,XP(I),YP(I),ZP(I),UX1(I),UY1(I),UZ1(I),CP1
LINE=LAST+1
LAST=LINE+54
PAGE=PAGE+1
IF(LINE-NOBP)>620,620,700
620 WRITE(6,605) PROB,PAGE
WRITE (6,601)
GO TO 606
700 IF (M1Y.EQ.0) GO TO 600
WRITE(6,605) PROB,PAGE
WRITE (6,603)
WRITE (6,601)
LINE=1
LAST=55
706 IF(NOBP-LAST)>707,710,710
707 LAST=NOBP
710 DO 715 I=LINE,LAST
CP2=1.- $(UX2(I)**2+UY2(I)**2+UZ2(I)**2)$ 
715 WRITE (6,611) I,XP(I),YP(I),ZP(I),UX2(I),UY2(I),UZ2(I),CP2
LINE=LAST+1
LAST=LINE+54
PAGE=PAGE+1
IF(LINE-NOBP)>720,720,800
720 WRITE(6,605) PROB,PAGE
WRITE (6,601)
GO TO 706
800 IF (M1Z.EQ.0) GO TO 825
WRITE(6,605) PROB,PAGE
WRITE (6,604)
WRITE (6,601)
LINE=1
LAST=55
806 IF(NOBP-LAST)>807,810,810
807 LAST=NOBP
810 DO 815 I=LINE,LAST
CP3=1.- $(UX3(I)**2+UY3(I)**2+UZ3(I)**2)$ 
815 WRITE (6,611) I,XP(I),YP(I),ZP(I),UX3(I),UY3(I),UZ3(I),CP3
LINE=LAST+1
LAST=LINE+54

```

```

PAGE=PAGE+1
IF(LINE-N0BP>820,820,825
820 WRITE(6,605) PROB,PAGE
WRITE (6,601)
GO TO 806
825 J = 1
826 IF (IREAD.EQ.0) GO TO 827
READ(5,26) UX4,VY4,UZ4
IF (EOF(5).NE. 0.) GO TO 900
GO TO 828
827 UX4=WS(J)
VY4 = WS(J+20)
UZ4 = WS(J+40)
828 CP=UX4**2+VY4**2+UZ4**2
IF(CP>900,900,830
C      R. COMPUTE FOURTH FLOW AND EDIT IT
830 LINE=1
LAST=51
WRITE(6,605) PROB,PAGE
831 FORMAT(19H00NSET FLOW    UX =,F7.3/15X,4HVY =,
1F7.3/15X,4HVZ =,F7.3)
WRITE (6,831) UX4,VY4,UZ4
WRITE (6,601)
835 IF(N0BP-LAST)>837,840,840
837 LAST=N0BP
840 DO 845 I=LINE,LAST
UXP=-UX4*UX1(I)-VY4*UX2(I)-UZ4*UX3(I)
UVP=-UX4*VY1(I)-VY4*VY2(I)-UZ4*VY3(I)
UZF=-UX4*UZ1(I)-VY4*UZ2(I)-UZ4*UZ3(I)
CP4= 1.-(UXP**2+UVP**2+UZF**2)/CP
845 WRITE (6,611) I,XP(I),YP(I),ZP(I),UXP,UVP,UZF,CP4
LINE=LAST+1
LAST=LINE+54
PAGE=PAGE+1
IF(LINE-N0BP>850,850,860
850 WRITE(6,605) PROB,PAGE
WRITE (6,601)
GO TO 835
860 J = J+1
GO TO 826
1000 WRITE(6,1001) J0,L,XP(J0),YP(J0),ZP(J0)
1001 FORMAT(16H00FF BODY POINT ,13,23H ON BOUNDARY OF QUAD ,13/
1           3H X=,F12.5,5X,2HY=,F12.5,5X,2HZ=,F12.5)
GO TO 530
900 CONTINUE
C      S. REWIND TAPES AND STOP
REWIND 03
REWIND 04
STOP 5
END
FUNCTION SQ2F(X1,X2,Y1,Y2,Z1,Z2)
X=X1-X2
Y=Y1-Y2
Z=Z1-Z2
RS=Z**2+Y**2+X**2
R=ABS(X)+ABS(Y)+ABS(Z)+ 1.0E-20
R=R+RS/R
R=.25*R+RS/R
R= R+RS/R
SQ2F= .25*R+RS/R
RETURN
END

```

APPENDIX UI - XYZPF SECTION PF6

```
PROGRAM PPF6(INPUT=128,TAPE16,OUTPUT=128,TAPE03,TAPE04,
1           TAPE5=INPUT,TAPE6=OUTPUT,TAPE3=TAPE03,TAPE4=TAPE04)
C
C XYZ POTENTIAL FLOW PROGRAM VERSION 4 SECTION 6
C COMPUTES VELOCITIES AND PRESSURE COEFFICIENTS FOR
C OFF BODY STREAMLINES
C
COMMON   XP(100),YP(100),ZP(100),UX1(100),WS(220)
1,UY1(100),UZ1(100),UX2(100),UY2(100),UZ2(100),UX3(100),UY3(100)
2,UZ3(100),UX(8),UY(8),UZ(8),S1(650),S2(650),S3(650),U1(3),U2(3),U3
3(3),PROB(15),XN(650),YN(650),ZN(650),TX(650),TY(650),TZ(650)
1,B(13000),XT(100),YT(100),ZT(100),AP(5),GM(4),SKY(100),
1,SK2(100),SKX(100),DX(100),DY(100),DZ(100),CP(100),TA(650)
EQUIVALENCE (KM,WS(211)),(KM,MK),(NP,WS(201)),(SVM,WS(210))
1,(Y2,Y3)
INTEGER SYM ,P
C      A. READ INPUT
      WRITE (6,5)
5 FORMAT(3I4,4F12.5)
8 FORMAT(3F12.5)
5 FORMAT(49H0XYZ POTENTIAL FLOW PROGRAM SECTION 6, VERSION 4 )
7 FORMAT (1X,9F12.5)
20 FORMAT(1H1,14,31H STREAMLINES TO BE COMPUTED AT ,14,10H STEPS OF
1 ,F8.4,28H T FOR AN ONSET VELOCITY OF ,3F8.4)
21 FORMAT(1X,15H STARTING POINTS/,3X,2HPT,5X,1HX,1HX,1HY,1HX,1HZ)
22 FORMAT(1X,14,3F12.5)
      READ (03) (PROB(I),I=1,15)
      WRITE (6,90) (PROB(I),I=1,15)
90 FORMAT(1H0,15A4)
C      B. READ THE PARAMETERS, T ARRAY AND SOURCE FROM TAPE 31
      READ (03) (WS(I),I=1,220)
      READ(03) (TX(I),TY(I),TZ(I),XN(I),YN(I),ZN(I),TA(I),I=1,NP)
      READ (03) SKIP
      IF ( WS(220) .EQ. 2. ) READ(03) SKIP
      READ (03) (S1(I),I=1,NP)
      READ (03) (S2(I),I=1,NP)
      READ (03) (S3(I),I=1,NP)
C      C. READ THE B ARRAY
      W2 = NP
      W2=(W2+11.0)/12.0
      NB = W2
      IS = 2
      IF=241
      DO 12 IP = 1,NB
      READ (04) P, (B(I),I=IS,IF)
      IS=IS+240
12 IF=IF+240
      RP(1)= .5
      RP(2) = .5
      RP(3) = 1.
      RP(4)=0.
      RP(5)=0.
      GM(1) = 1./6.
      GM(2) = 1./3.
      GM(3) = 1./3.
83 READ (5,6) NOBP,NST,IEND,DT,UX1,UY1,UZ1
      USO=UX1**2+UY1**2+UZ1**2
      NOB=NOBP
      DO 10 I=1,NOB
      READ(5, 8) XP(I),YP(I),ZP(I)
```

```

      IF (EOF(5) .EQ. 0.) GO TO 10
      NOBP=1-1
      WRITE(6,9) NOBP,NOB
      9 FORMAT(1HO,15, 28H STREAMLINES SPECIFIED NOT ,13)
      GO TO 11
10 CONTINUE
11 CONTINUE
      WRITE(16) NOBP,NST,1END,UX1,UY1,UZ1
      WRITE(6,20) NOBP,NST,DT,UX1,UY1,UZ1
      WRITE(6,21)
      WRITE(6,22) (I,XP(I),YP(I),ZP(I), I=1,NOBP)
C      NOBP - NUMBER OF STREAMLINES TO BE TRACED.
C      NST - NUMBER OF STATIONS AT WHICH STREAMLINES SHOULD BE COMPUTED.
      DO 15 I=1,NOBP
      XT(I) = XPC(I)
      YT(I) = YPC(I)
      ZT(I) = ZPC(I)
      SKX(I)=0.
      SKY(I) = 0.
15 SKZ(I) = 0.
      ITG=0
      IRK=5
98 K=1
      P = 1
      J=1
      DO 100 I=1,NOBP
C          D. SET THE PARTIAL VELOCITY TO THE FREE STREAM VELOCITY
      UX1(I)=-1.0
      UY1(I)=0.
      UZ1(I)=0.
      UX2(I)=0.
      UY2(I)=-1.0
      UX3(I)=0.0
      UY3(I)=0.0
      UZ3(I)=-1.0
100 UZ2(I)=0.
C          E. START LOOP OVER THE QUADS.
295 J=2
C          F1 PICK UP QUAD. INFORMATION
296 X1=B(J)
      Y1=B(J+1)
      X2=B(J+2)
      Y2=B(J+3)
      X3=B(J+4)
      Y3=B(J+5)
      X4=B(J+6)
      Y4=B(J+7)
      XC=TX(K)
      YC=TY(K)
      ZC=TZ(K)
      A =TA(K)
      XX=B(J+8)
      YX=B(J+9)
      ZX=B(J+10)
      XY=B(J+11)
      YY=B(J+12)
      ZY=B(J+13)
C          F2 COMPUTE LENGTH OF SIDES OF QUAD.
      D12=SQ2F(X1,X2,Y1,Y2,0.,0.)
      D23=SQ2F(X2,X3,Y2,Y3,0.,0.)
      D34=SQ2F(X3,X4,Y3,Y4,0.,0.)
      D41=SQ2F(X4,X1,Y4,Y1,0.,0.)
C          F3 COMPUTE SLOPE OF SIDES

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        IF(X2-X3)305,300,305
300 C123=1.
      GO TO 310
305 CM23=(Y2-Y3)/(X2-X3)
      C123=0.
310 IF(X3-X4)315,311,315
311 C134=1.
      GO TO 320
315 CM34=(Y4-Y3)/(X4-X3)
      C134=0.
320 IF(X4-X1)325,321,325
321 C141=1.
      GO TO 330
325 CM41=(Y1-Y4)/(X1-X4)
      C141=0.
330 IF(X1-X2)335,331,335
331 C112=1.
      GO TO 340
335 CM12=(Y2-Y1)/(X2-X1)
      C112=0.
C      F4 COMPUTE QUADRAPOLE MOMENTS
340 C1XX=B(J+17)
      C1XY=B(J+18)
      C1YY=B(J+19)
C      F5 COMPUTE SIN AND COS OF SLOPE ANGLE FOR EACH SIDE
CY12=(Y2-Y1)/D12
CY23=(Y3-Y2)/D23
CY34=(Y4-Y3)/D34
CY41=(Y1-Y4)/D41
CX12=(X1-X2)/D12
CX23=(X2-X3)/D23
CX34=(X3-X4)/D34
CX41=(X4-X1)/D41
C      F6 COMPUTE MAX DIAGINAL
ST=SQ2F(X1,X3,Y1,Y3,0.,0.)
ST2=SQ2F(X2,X4,Y2,Y4,0.,0.)
IF(ST-ST2)341,342,342
341 ST=ST2
C      G. START LOOP OVER THE OFF BODY POINTS
342 DO 330 JQ=1,NQBP
      IS=1
      X00=XP(JQ)
      Y00=YP(JQ)
      Z00=ZF(JQ)
      J1=1
      345 RPO=SQ2F(X0,X00,Y0,Y00,Z0,Z00)
C      H. DETERMIN METHOD
      IF(RPO-ST*4)350,350,460
350 X=(X00-X0)*XX+(Y00-Y0)*YX+(Z00-Z0)*ZX
      Y=(X00-X0)*XY+(Y00-Y0)*YY+(Z00-Z0)*ZY
      Z=(X00-X0)*ZN(K)+((Y00-Y0)*VN(K)+(Z00-Z0)*ZN(K))
      IF(RPO-ST*2.5)355,355,400
C      ! COMPUTE INDUSED VELOCITY BY EXACT METHOD
355 R1=SQ2F(X,X1,Y,Y1,Z,0.)
      R2=SQ2F(X,X2,Y,Y2,Z,0.)
      R3=SQ2F(X,X3,Y,Y3,Z,0.)
      R4=SQ2F(X,X4,Y,Y4,Z,0.)
      CLA1=RLOG((R1+R2-D12)/(R1+R2+D12))
      CLA2=RLOG((R2+R3-D23)/(R2+R3+D23))
      CLA3=RLOG((R3+R4-D34)/(R3+R4+D34))
      CLA4=RLOG((R4+R1-D41)/(R4+R1+D41))
      TUX=CY12*CLA1+CY23*CLA2+CY34*CLA3+CY41*CLA4

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TUV=CX12*CLR1+CX23*CLR2+CX34*CLR3+CX41*CLR4
TUZ=0.
IF(ABS(Z)-.001*ST)375,361,361
361 ZSQ=Z**2
E1=ZSQ+(X-X1)**2
E2=ZSQ+(X-X2)**2
E3=ZSQ+(X-X3)**2
E4=ZSQ+(X-X4)**2
H1=(Y-Y1)*(X-X1)
H2=(Y-Y2)*(X-X2)
H3=(Y-Y3)*(X-X3)
H4=(Y-Y4)*(X-X4)
IF(C112)363,363,364
363 WS1=(CM12*E1-H1)/(Z*R1)
WS2=(CM12*E2-H2)/(Z*R2)
AT1=ATAN(WS1)
AT2=ATAN(WS2)
TUZ=AT1-AT2
364 IF(C123)366,366,367
366 AT3=ATAN((CM23*E2-H2)/(Z*R2))
AT4=ATAN((CM23*E3-H3)/(Z*R3))
TUZ=TUZ+AT3-AT4
367 IF(C134)368,368,369
368 AT5=ATAN((CM34*E3-H3)/(Z*R3))
AT6=ATAN((CM34*E4-H4)/(Z*R4))
TUZ=TUZ+AT5-AT6
369 IF(C141)370,370,375
370 AT7=ATAN((CM41*E4-H4)/(Z*R4))
AT8=ATAN((CM41*E1-H1)/(Z*R1))
TUZ=TUZ+AT7-AT8
375 GO TO 450
C J. COMPUTE INDUSED VELOCITY BY QUADRAPOLE METHOD
400 RPQ3=RPQM**3
RPQ7=(RPQ3**2)*RPQ
WS1=X/RPQ3
XSQ=X**2
YSQ=Y**2
ZSQ=Z**2
PS=VSQ+ZSQ-4.*XSQ
QS=XSQ+ZSQ-4.*YSQ
WS2=X*(9.*PS+30.*XSQ)/RPQ7
WS3=3.*Y*PS/RPQ7
WS4=3.*X*QS/RPQ7
TUX=R*WS1-C1XY*WS3-C1XX*WS2-C1YY*WS4
WS1=Y/RPQ3
WS2=Y*(9.*QS+30.*YSQ)/RPQ7
TUV=RY*WS1-C1XY*WS3-C1XY*WS4-C1YY*WS2
TUZ=Z*(R/RPQ3-3.*C1XX*PS-5.*C1XY*XY+C1YY*QS)/RPQ7
450 UX(J1)=TUX*XX+TUV*XY+TUZ*XN(K)
UY(J1)=TUX*YX+TUV*YY+TUZ*YN(K)
UZ(J1)=TUX*ZX+TUV*ZY+TUZ*ZN(K)
GO TO 470
C K. COMPUTE INDUSED VELOCITY BY MONPOLE METHOD
460 ARPQ3=R/(RPQ**3)
UX(J1)=(XC0-XC)*ARPQ3
UY(J1)=(YC0-YC)*ARPQ3
UZ(J1)=(ZC0-ZC)*ARPQ3
C L. REFLECT OFF BODY POINT IN PLANE OF SYMETRY
470 GO TO(480,485,490,495,500,505,510,515),15
480 U1(J1)=UX(J1)
U2(J1)=UY(J1)
U3(J1)=UZ(J1)

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U1(J1+1)=UY(1)
U2(J1+1)=UY(1)
U3(J1+1)=UY(1)
U1(J1+2)=UZ(1)
U2(J1+2)=UZ(1)
U3(J1+2)=UZ(1)
IF(SYM) 525,525,481
481 IS=2
C          X2 SYMETRY
YCQ=-YCO
GO TO 345
485 IF(SYM-1)517,517,486
C          XY SYMETRY
486 IS=3
ZCO=-ZCO
GO TO 345
490 IS=4
YCQ=-YCO
GO TO 345
495 IF(SYM-2)516,516,496
C          YZ SYMETRY
496 IS=5
XCQ=-XCO
GO TO 345
500 IS=6
YCQ=-YCO
GO TO 345
505 IS=7
ZCO=-ZCO
GO TO 345
510 IS=8
YCQ=-YCO
GO TO 345
C          M. ADD CONTRIBUTIONS OF ALL REFLECTIONS
515 U1(J1)=U1(J1)+UX(8)+UX(7)+UX(6)+UX(5)
U2(J1)=U2(J1)-UX(8)+UX(7)+UX(6)-UX(5)
U3(J1)=U3(J1)-UX(8)-UX(7)+UX(6)+UX(5)
U1(J1+1)=U1(J1+1)-UY(8)+UY(7)+UY(6)-UY(5)
U2(J1+1)=U2(J1+1)+UY(8)+UY(7)+UY(6)+UY(5)
U3(J1+1)=U3(J1+1)+UY(8)-UY(7)+UY(6)-UY(5)
U1(J1+2)=U1(J1+2)-UZ(8)-UZ(7)+UZ(6)+UZ(5)
U2(J1+2)=U2(J1+2)+UZ(8)-UZ(7)+UZ(6)-UZ(5)
U3(J1+2)=U3(J1+2)+UZ(8)+UZ(7)+UZ(6)+UZ(5)
516 U1(J1)=U1(J1)+UX(4)+UX(3)
U2(J1)=U2(J1)+UX(4)-UX(3)
U3(J1)=U3(J1)-UX(4)-UX(3)
U1(J1+1)=U1(J1+1)+UY(4)-UY(3)
U2(J1+1)=U2(J1+1)+UY(4)+UY(3)
U3(J1+1)=U3(J1+1)-UY(4)+UY(3)
U1(J1+2)=U1(J1+2)-UZ(4)-UZ(3)
U2(J1+2)=U2(J1+2)-UZ(4)+UZ(3)
U3(J1+2)=U3(J1+2)+UZ(4)+UZ(3)
517 U1(J1)=U1(J1)+UX(2)
U2(J1)=U2(J1)-UX(2)
U3(J1)=U3(J1)+UX(2)
U1(J1+1)=U1(J1+1)-UY(2)
U2(J1+1)=U2(J1+1)+UY(2)
U3(J1+1)=U3(J1+1)-UY(2)
U1(J1+2)=U1(J1+2)+UZ(2)
U2(J1+2)=U2(J1+2)-UZ(2)
U3(J1+2)=U3(J1+2)+UZ(2)
525 UX1(J0)=UX1(J0)+U1(1)*S1(P)

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```

UY1(JQ)=UY1(JQ)+U1(2)*S1(P)
UZ1(JQ)=UZ1(JQ)+U1(3)*S1(P)
UX2(JQ)=UX2(JQ)+U2(1)*S2(P)
UY2(JQ)=UY2(JQ)+U2(2)*S2(P)
UZ2(JQ)=UZ2(JQ)+U2(3)*S2(P)
UX3(JQ)=UX3(JQ)+U3(1)*S3(P)
UY3(JQ)=UY3(JQ)+U3(2)*S3(P)
530 UZ3(JQ)=UZ3(JQ)+U3(3)*S3(P)
C      N. END OF LOOP OVER OFF BODY POINTS
585 P=P+1
K=K+1
J=J+20
IF(K-NP .GT.296,296,599
C      P. END OF LOOP OVER QUADS
599 H=AP(IRK)*DT
DO 730 I = 1,NOBP
63 FORMAT(2X,13,3F12.5,9X,4F12.5)
DX(I)=- $(UX1*UX1(I)+UY1*UX2(I)+UZ1*UX3(I))$ 
DY(I)=- $(UX1*UY1(I)+UY1*UY2(I)+UZ1*UY3(I))$ 
730 DZ(I)=- $(UX1*UZ1(I)+UY1*UZ2(I)+UZ1*UZ3(I))$ 
IF(IRK.EQ.5) GO TO 900
IF(IRK.EQ.4) GO TO 800
DO 750 I=1,NOBP
XP(I)=XT(I)+DX(I)*H
YP(I)=YT(I)+DY(I)*H
ZP(I)=ZT(I)+DZ(I)*H
SKX(I)=SKX(I)+GM(IRK)*DX(I)
SKY(I)=SKY(I)+GM(IRK)*DY(I)
750 SKZ(I)=SKZ(I)+GM(IRK)*DZ(I)
IRK = IRK + 1
GO TO 98
800 H=DT
DO 830 I=1,NOBP
DX(I)=- $(UX1*UX1(I)+UY1*UY2(I)+UZ1*UX3(I))$ 
DY(I)=- $(UX1*UY1(I)+UY1*UY2(I)+UZ1*UY3(I))$ 
DZ(I)=- $(UX1*UZ1(I)+UY1*UZ2(I)+UZ1*UZ3(I))$ 
XP(I)=XT(I)+H*DX(I)/6.+SKX(I)*H
XT(I)=XP(I)
YP(I)=YT(I)+H*DY(I)/6.+SKY(I)*H
YT(I)=YP(I)
ZP(I)=ZT(I)+H*DZ(I)/6.+SKZ(I)*H
ZT(I)=ZP(I)
SKX(I)=0.
SKY(I) = 0.
830 SKZ(I) = 0.
IRK = 5
GO TO 98
900 IRK = 1
DO 905 I=1,NOBP
DSQ=DX(I)**2+DY(I)**2+DZ(I)**2
CP(I)=1.-DSQ/USQ
905 CONTINUE
WRITE(6,61) ITC
61 FORMAT(6HO STEP,14/)
WRITE(6,62)
62 FORMAT(3X,4HLINE,5X,1HX,11X,1HY,11X,1HZ,20X,2HUX,10X,2HUY,
1      10X,2Huz,10X,2HCP)
WRITE(6,63) (1,XP(I),YP(I),ZP(I),DX(I),DY(I),DZ(I),CP(I),I=1,NOBP)
WRITE(16) (XP(I),YP(I),ZP(I), I=1,NOBP)
IF(ITC.EQ.NST) GO TO 910
ITC=ITC+1
GO TO 599

```

```
910 IF(IEND.EQ.0 ) GO TO 93
REWIND 03
REWIND 04
ENDFILE 16
REWIND 16
STOP 6
END
FUNCTION SQ2F(X1,X2,Y1,Y2,Z1,Z2)
X=X1-X2
Y=Y1-Y2
Z=Z1-Z2
RS=Z**2+Y**2+X**2
R=ABS(X)+ABS(Y)+ABS(Z)+ 1.0E-20
R=.3422*(R+(RS+RS)/R)
R= R+RS/R
SQ2F= .25*R+RS/R
RETURN
END
```

APPENDIX U11 - XYZPF SECTION PF7

```
PROGRAM PPP7(TAPE7, INPUT=128, OUTPUT=128, TAPE5=INPUT, TAPE17,
1TAPE6=OUTPUT, TAPE03, TAPE04, TAPE3=TAPE03, TAPE4=TAPE04)

C
C XYZ POTENTIAL FLOW PROGRAM VERSION 4 AND VERSION 5 SECTION 7
C COMPUTES VELOCITIES AND PRESSURE COEFFICIENTS FOR
C ON BODY STREAMLINES
C

COMMON X(658),Y(658),Z(658),XN(650),YN(650)
1,ZN(650),UX1(650),UY1(650),UZ1(650),UX2(650),UY2(650)
2,UZ2(650),UX3(650),UY3(650),UZ3(650),XC1(658),YC1(658)
3,XC2(658),YC2(658),XC3(658),YC3(658),XC4(658),YC4(658),
4X3(658),Y3(658),Z3(658),X4(658),Y4(658),Z4(658)
DIMENSION XL(150),YL(150),ZL(150),UX(150),UY(150),UZ(150),
1CP(150),GK1(150),GK2(150),H2(150),STML(150),URSS(150),NUURD(150)
5,DMX(650),PROB(15),YC3(658),SF(5),XCR(5),YCR(5) ,
7NSP(50),WS(220),XST(50),YST(50),ZST(50)
EQUIVALENCE (WS(201),NP),(YC3,YC2)
READ(03)(PROB(1), I=1,15)
READ(03)(WS(1), I=1,220)
READ(03)(X(1),Y(1),Z(1),XN(1),YN(1),ZN(1),
1SK1P, I=1,NP)
READ(03)SK1P
IVER =WS(220)
WRITE(6,5) IVER
5 FORMAT(46HOXYZ POTENTIAL FLOW PROGRAM SECTION 7, VERSION ,12)
IF (IVER.EQ.5) READ(03) SKIP
READ(03)SK1P
READ(03)SK1P
READ(03)SK1P
READ(03)(UX1(1),UY1(1),UZ1(1),I=1,NP)
READ(03)(UX2(1),UY2(1),UZ2(1),I=1,NP)
READ(03)(UX3(1),UY3(1),UZ3(1),I=1,NP)
REWIND 03
NE=(NP+11)/12
DO 80 I=1,NE
IFN=I*12
IS=IFN-11
REPE(04) 0,(XC1(J),YC1(J),XC2(J),
1YC2(J),XC3(J),YC3(J),XC4(J),YC4(J),X3(J),Y3(J),
2Z3(J),X4(J),Y4(J),Z4(J),(SKIP,K=1,7),J=IS,IFN)
NO=0
IF (NO.NE.IS) GO TO 450
80 CONTINUE
REWIND 04
DO 90 I=1,NP
D1=(XC1(I)**2+YC1(I)**2)*1.01
D2=(XC2(I)**2+YC2(I)**2)*1.01
D3=(XC3(I)**2+YC3(I)**2)*1.01
D4=(XC4(I)**2+YC4(I)**2)*1.01
90 DMX(I)=AMAX1(D1,D2,D3,D4)
11 FORMAT(3F12.4,3I4,F12.4)
12 FORMAT(3F12.4, 14)
MID=75
100 READ(5,11) UX1,UY1,UZ1,NLIN,MAXJ,WRITE,AMACH
IF (EOF(5).NE.0.) LIN=0
MXJ=MAXJ
IF ( MAXJ .LE. 0 .OR. MAXJ .GT. NF/2) MAXJ = NF/2
MINJ=MID-MAXJ
MAXJ=MID+MAXJ
IF (MAXJ.GT.MID*2) MAXJ=MID*2
```

```

      IF ( MINJ .LT. 1) MINJ = 1
      WRITE(7) NLIN
      WRITE(17) NLIN,UXI,UVI,UZI
      IF(NLIN.LE.0) GO TO 550
      WRITE(6,30) (PROB(I),I=1,15)
      WRITE(6,34) UXI,UVI,UZI,NLIN,MXJ,IWRITE,AMACH
34 FORMAT(34H00N BODY STREAMLINES - INPUT DATA /6H UXI =,F10.5/
       16H UVI =,F10.5/6H UZI =,F10.5/6H NLIN=,I10/
       2 6H JMAX=,I10,/,,SH IWRITE=,I10,/,,9H MACH NO.=,F10.5)
      WRITE(6,38)
36 FORMAT(27H00STREAMLINE STARTING POINTS/5H LINE,11X,1HX,12X,1HY,
       1 12X,1HZ,10X,3HNSP)
      LIN=MLIN
      DO 45 I=1,LIN
      READ(5,12) XST(I),YST(I),ZST(I),NSP(I)
      IF (EOF(5).EQ.0.) GO TO 45
      NLIN=I-1
      WRITE(6,42) NLIN,LIN
42 FORMAT(1H0,15,28H STREAMLINES SPECIFIED NOT ,13)
      IF(NLIN.LE.0) GO TO 550
      GO TO 48
45 WRITE(6,46) I,XST(I),YST(I),ZST(I),NSP(I)
46 FORMAT(1X,13,2X,3F13.5,19)
48 CONTINUE
      USQ=UXI**2+UVI**2+UZI**2
      IF (AMACH .EQ. 0.) GO TO 1130
C *** COMPUTE CRITICAL MACH NO.
      USD = 0
      DO 1100 I=1,NP
      US = ((UXI*UXI(I))+UVI*UVI(I)+UZI*UZI(I))**2 +
1      ((UXI*UVI(I))+UVI*UVI(I)+UZI*UZI(I))**2 +
2      ((UXI*UZI(I))+UVI*UVI(I)+UZI*UZI(I))**2
      IF (US .GT. USD) USD = US
1100 CONTINUE
      U = SQRT(USD/USQ)
      CMNA = 1./U
      DO 1110 I=1,3
      CMNE = (((CMNA**2+5.)/6.)**1.75)/U
      CMNC = (((CMNE**2+5.)/6.)**1.75)/U
1110 CMNA = (CMNA*CMNC-CMNE**2)/(CMNA+CMNC-2.*CMNE)
      WRITE(6,1120) CMNA
1120 FORMAT(21H CRITICAL MACH NO. =,F5.3)
1130 CONTINUE
C START LOOP OVER STREAMLINES
      DO 400 LL=1,NLIN
      GIFT=1
101 JI=1
      RF=1
      UX(MJD)=0.
      UV(MJD)=0.
      UZ(MJD)=0.
      CP(MJD)=0
      H2(MJD)=1.
      GM1(MJD)=0.
      GK2(MJD)=0.
      STM(MJD)=0
102 ND=NSP(LL)
      LND=ND
      XL(MJD)=XST(LL)
      YL(MJD)=YST(LL)
      ZL(MJD)=ZST(LL)
      J=MJD

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JL=J
C SEPARATE CALCULATION OF SECOND
C POINT FROM MAIN LOOP
XLT=(XL(J)-X(NQ))*X3(NQ)+(YL(J)-Y(NQ))*Y3(NQ)
1 +(ZL(J)-Z(NQ))*Z3(NQ)
YLT=(XL(J)-X(NQ))*X4(NQ)+(YL(J)-Y(NQ))*Y4(NQ)
1 +(ZL(J)-Z(NQ))*Z4(NQ)
XL(J)=XLT*X3(NQ)+YLT*X4(NQ)+ZL(NQ)
YL(J)=XLT*Y3(NQ)+YLT*Y4(NQ)+ZL(NQ)
ZL(J)=XLT*Z3(NQ)+YLT*Z4(NQ)+ZL(NQ)
105 IQT=MOD(NQ,4) + 1
GO TO 630,600,610,620 IQT
600 NR=NQ+1
NU=NQ+2
GO TO 107
610 NR=NQ+2
NU=NQ-1
GO TO 107
620 NR=NQ-2
NU=NQ+1
GO TO 107
630 NR=NQ-1
NU=NQ-2
107 UXI=-UX1*UX1(NQ)+UY1*UX2(NQ)+UZ1*UX3(NQ)
UYI=-UX1*UY1(NQ)+UY1*UY2(NQ)+UZ1*UY3(NQ)
UZI=-UX1*UZ1(NQ)+UY1*UZ2(NQ)+UZ1*UZ3(NQ)
UXF=-UX1*UX1(NR)+UY1*UX2(NR)+UZ1*UX3(NR)
UYF=-UX1*UY1(NR)+UY1*UY2(NR)+UZ1*UY3(NR)
UZF=-UX1*UZ1(NR)+UY1*UZ2(NR)+UZ1*UZ3(NR)
UXU=-UX1*UX1(NU)+UY1*UX2(NU)+UZ1*UX3(NU)
UYU=-UX1*UY1(NU)+UY1*UY2(NU)+UZ1*UY3(NU)
UZU=-UX1*UZ1(NU)+UY1*UZ2(NU)+UZ1*UZ3(NU)
C TRANSFORM VELOCITIES TO QUAD SYSTEM
UU=UX0*X3(NQ)+UY0*Y3(NQ)+UZ0*Z3(NQ)
UU=UX0*X4(NQ)+UY0*Y4(NQ)+UZ0*Z4(NQ)
CSR=1./((X(NQ)*X(NR))+YN(NQ)*VN(NR)+ZN(NQ)*ZN(NR))
UT=UXR*X3(NR)+UYR*Y3(NR)+UZR*Z3(NR)
UT=(UYR*X4(NR)+UYR*Y4(NR)+UZR*Z4(NR))/CSR
XXR=(X3(NR)*X3(NQ)+Y3(NR)*Y3(NQ)+Z3(NR)*Z3(NQ))
XYR=(X4(NR)*X3(NQ)+Y4(NR)*Y3(NQ)+Z4(NR)*Z3(NQ))
UR=UT*XXR+UT*XYR
YXR=(X3(NR)*X4(NQ)+Y3(NR)*Y4(NQ)+Z3(NR)*Z4(NQ))
YYR=(X4(NR)*X4(NQ)+Y4(NR)*Y4(NQ)+Z4(NR)*Z4(NQ))
UR=UT*YXR+UT*YYR
UU=UXU*X3(NU)+UYU*Y3(NU)+UZU*Z3(NU)
CSU=(X(NQ)*X(NU)+YN(NQ)*VN(NU)+ZN(NQ)*ZN(NU))/CSU
UU=(UXU*X4(NU)+UYU*Y4(NU)+UZU*Z4(NU))/CSU
C FIND RELATIVE COORDINATES OF NEIGHBORING QUADS
XD=X(NR)-X(NQ)
YD=Y(NR)-Y(NQ)
ZD=Z(NR)-Z(NQ)
XT=XD*X3(NR)+YD*Y3(NR)+ZD*Z3(NR)
YTT=XD*X4(NR)+YD*Y4(NR)+ZD*Z4(NR)
ZT=XD*X3(NR)+YD*Y3(NR)+ZD*Z3(NR)
YT=(-4*SORT(YTT**2+ZT**2)+YTT*CSR+YTT)*CSR*.16666667
XR=XT*X2R+YT*XVR
YR=XT*YXR+YT*YVR
XD=X(NU)-X(NQ)
YD=Y(NU)-Y(NQ)
ZD=Z(NU)-Z(NQ)
YU=XD*X3(NQ)+YD*Y3(NQ)+ZD*Z3(NQ)
YT=XD*X4(NQ)+YD*Y4(NQ)+ZD*Z4(NQ)

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ZT=XD*XN(NQ)+YD*YN(NQ)+ZD*ZN(NQ)
YU=(4.*SQR(TT**2+ZT**2)+YT/CSU+YT)*.16666667
C FIND COEFFICIENTS OF VELOCITY FUNCTIONS
DEN=1./((XR*YU-XU*YR))
U1=((UR-UQ)*YU-(UU-UQ)*YR)*DEN
U2=-((UR-UQ)*XU-(UU-UQ)*XR)*DEN
V1=((UR-UQ)*YU-(UU-UQ)*YR)*DEN
V2=-((UR-UQ)*XU-(UU-UQ)*XR)*DEN
C FIND VELOCITY AT STREAMLINE POINT
USL=UQ+U1*XLT+U2*YLT
VSL=UQ+V1*XLT+V2*YLT
UXP=USL*X3(NQ)+USL*X4(NQ)
VNP=USL*Y3(NQ)+USL*Y4(NQ)
UCP=USL*Z3(NQ)+USL*Z4(NQ)
C FIND GEODESIC CURVATURES GK1, GK2
USOC=USL**2+VSL**2
DEN=USOC*SORT(USOC)
GK1P=(USL*(U1*U2-UQ*U2)-USL*(U1*U1-UQ*U1))/DEN
GK2P=(USL*(U2*U1+UQ*U2)-USL*(U1*U1+UQ*U2))/DEN
C FIND LOCAL STREAM FUNCTION
CXW=(U1*U2**2+UQ*U2**2)/USOC
CYV=U2-UQ*(U1+U2)/USOC
COX=U2-CYV-U1
COY=XLT*YQ-VLT*UQ-CXW*XLT-CYV*YLT-CXX*XLT**2
C FIND STREAM FUNCTION AT CORNER POINTS
XCR(1)=XC1(NQ)
XCR(2)=XC2(NQ)
XCR(3)=XC3(NQ)
XCR(4)=XC4(NQ)
XCR(5)=XCR(1)
YCR(1)=YC1(NQ)
YCR(2)=YC2(NQ)
YCR(3)=YC3(NQ)
YCR(4)=YC4(NQ)
YCR(5)=YCR(1)
DO 110 N=1,4
110 SF(N)=CO-UQ*XCR(N)+UQ*YCR(N)+CXW*XCR(N)*YCR(N)+CYV*YCR(N)**2
     +CXX*XCR(N)**2
SF(5)=SF(1)
TEST=0
DO 120 N=1,4
120 IF (<SF(N)*SF(N+1)>.GE. 0.) GO TO 120
XM=(XCR(N)+XCR(N+1))*5
VM=(YCR(N)+YCR(N+1))*5
C FIND INTERSECTION WITH SIDE OF QUAD.
SF=M-UQ*XM+UQ*VM+CXW*XM*VM+CYV*VM**2+CXX*XM**2
AC=2.*SF(N)-2.*SFM+SF(N+1)
BC=3.*SF(N)-4.*SFM+SF(N+1)
IF (<AC>.EQ. 0) GO TO 113
SR=SORT(BC**2-4.*AC*SF(N))
TP=(BC+SR)/(2.*AC)
IF (<TP>LE. 1. AND. TP >= 0.) GO TO 115
TP=(BC-SR)/(2.*AC)
GO TO 115
113 IF (<BC>.EQ. 0) GO TO 120
TP=SF(N)/BC
115 XNP=(1.-TP)*XCR(N)+TP*XCR(N+1)
VNP=(1.-TP)*YCR(N)+TP*YCR(N+1)
TESTP=((XNP-XLT)*UQ+(VNP-YLT)*UQ)*DIRT
IF (<TESTP>LE. TEST) GO TO 120
TEST=TESTP
XNT=XNP

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YNT=VNP
120 CONTINUE
IF( TEST .LE. 0 ) GO TO 280
C AVERAGE LAST VELOCITY AND CURVATURE
UX(J)=(UX(J)+UXP)*AF
UY(J)=(UY(J)+UYP)*AF
UZ(J)=(UZ(J)+UZP)*AF
GK1(J)=(GK1(J)+GK1P)*AF
GK2(J)=(GK2(J)+GK2P)*AF
H2(J)=H2(JL)*(2.-GK1(JL)*(STML(J)-STML(JL)))/(2.+GK1(J)*(1.-STML(J)-STML(JL)))
CP(J)=1.-(UX(J)**2+UY(J)**2+UZ(J)**2)/US0
URBS(J)=SQRT(1.-CP(J))
C COMPUTE VELOCITY AT NEXT POINT
NQUAD(J)=NO
JL=J
J=J+JI
USL=UQ+XNT*U1+YNT*U2
USL=UQ+XNT*U1+YNT*U2
UX(J)=USL*X3(N0)+USL*Z4(N0)
UY(J)=USL*Y3(N0)+USL*Y4(N0)
UZ(J)=USL*Z3(N0)+USL*Z4(N0)
C COMPUTE GEODESIC CURVATURES
USQD=USL**2+USL**2
DEN=USQD*SORT(USQD)
GK1(J)=(USL*(USL*U2-USL*U2)-USL*(USL*U1-USL*U1))/DEN
GK2(J)=(USL*(USL*U1+USL*U2)-USL*(USL*U1+USL*U2))/DEN
CORD=SORT((XNT-XLT)**2+(YNT-YLT)**2)*PI/RT
STML(J)=STML(JL)+CORD
C COMPUTE H2
H2(J)=H2(JL)*(2.-CORD*GK1(JL))/(2.+CORD*GK1(J))
CP(J)=1.-USQD/US0
URBS(J)=SQRT(1.-CP(J))
RF=.5
LN0=NO
XL(J)=XNT*X3(N0)+YNT*X4(N0)+Y(N0)
YL(J)=XNT*Y3(N0)+YNT*Y4(N0)+Y(N0)
ZL(J)=XNT*Z3(N0)+YNT*Z4(N0)+Z(N0)
IF( J .LE. MINJ .OR. J .GE. MAXJ ) GO TO 280
C FIND NEXT QUAD.
I=1
250 NO=1
IF(I .EQ. LN0) GO TO 290
TEST=(XL(J)-XC1(1))**2+(YL(J)-YC1(1))**2+
1(ZL(J)-ZC1(1))**2-DMY(I)
IF(TEST .GT. 0.) GO TO 280
DS1=(XC1(1)-XC2(1))**2+(YC1(1)-YC2(1))**2
DS2=(XC2(1)-XC3(1))**2+(YC2(1)-YC3(1))**2
DS3=(XC3(1)-XC4(1))**2+(YC3(1)-YC4(1))**2
DS4=(XC4(1)-XC1(1))**2+(YC4(1)-YC1(1))**2
XLT=XL(J)-XC1(1)*X3(1)+YL(J)-YC1(1)*Y3(1)+1
(ZL(J)-ZC1(1))*Z3(1)
YLT=XL(J)-XC1(1)*Y4(1)+YL(J)-YC1(1)*Y4(1)+1
(ZL(J)-ZC1(1))*Z4(1)
ZLT=XL(J)-XC1(1)*XN(1)+YL(J)-YC1(1)*YN(1)+1
(ZL(J)-ZC1(1))*ZN(1)
ZS0=ZLT**2
TEST=ZS0-.1*DMY(I)
IF(TEST .GT. 0.) GO TO 280
RC1=SORT(ZS0+(XLT-XC1(1))**2+(YLT-YC1(1))**2)
RC2=SORT(ZS0+(XLT-XC2(1))**2+(YLT-YC2(1))**2)

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RC3=SQRT(ZSQ+(XLT-XC3(I))**2+(YLT-YC3(I))**2)
RC4=SQRT(ZSQ+(XLT-XC4(I))**2+(YLT-YC4(I))**2)
TEST= ((RC1+RC2)**2)-DS1 *1.21
IF(TEST.LT.0.) GO TO 105
TEST= ((RC2+RC3)**2)-DS2 *1.21
IF(TEST.LT.0.) GO TO 105
TEST= ((RC3+RC4)**2)-DS3 *1.21
IF(TEST.LT.0.) GO TO 105
TEST= ((RC4+RC1)**2)-DS4 *1.21
IF(TEST.LT.0.) GO TO 105
280 I=I+1
IF(I.LE.NP) GO TO 250
282 IF (DIRT.LT.0.) GO TO 285
DIRT=-1.
J1=-1
JMAX=J
GO TO 102
285 JMIN=J
SS=STML(JMIN)
DO 290 J=JMIN,JMAX
290 STML(J)=STML(J)-SS
JMN=JMIN+1
JMX=JMAX-2
RF=1.
L=JMN
WRITE(6,30)(PROB(I),I=1,15)
30 FORMAT(1H1,15R4)
WRITE(6,20)UX1,UV1,UZ1
20 FORMAT( 18HO CONSET FLOW, UX1=F6.3,2X,4HUV1=F6.3,2X,4HUZ1=F6.3)
WRITE(6,50) LL,NSP(LL),XST(LL),YST(LL),ZST(LL)
50 FORMAT(11HO LINE NO. ,12,31H PASSING THROUGH QUADRILATERAL ,13,
1           28H WITH STARTING POINT,      X=F12.5,2X,2HV=F12.5,2X,
2           2HZ=F12.5 //)
IF (JMIN.LE.MINJ .OR. JMAX.GE.MAXJ) WRITE(6,55)
65 FORMAT(3SH PROBABLE ERROR - LINE IS VERY LONG)
DO 330 J=JMN,JMX
IF (< (STML(J+2)-STML(L-1)),LT. 8.* (STML(J+1)-STML(L))) GO TO 320
WRITE(6,310) XL(L),YL(L),ZL(L),XL(J+1),YL(J+1),ZL(J+1)
310 FORMAT(14H POINT DELETED ,10X,3F12.5,10X,3F12.5)
STML(L)=(RF*STML(L)+STML(J+1))/(RF+1.)
XL(L)=(RF*XL(L)+XL(J+1))/(RF+1.)
YL(L)=(RF*YL(L)+YL(J+1))/(RF+1.)
ZL(L)=(RF*ZL(L)+ZL(J+1))/(RF+1.)
UX(L)=(RF*UX(L)+UX(J+1))/(RF+1.)
UY(L)=(RF*UY(L)+UY(J+1))/(RF+1.)
UZ(L)=(RF*UZ(L)+UZ(J+1))/(RF+1.)
GK1(L)=(RF*GK1(L)+GK1(J+1))/(RF+1.)
GK2(L)=(RF*GK2(L)+GK2(J+1))/(RF+1.)
H2(L)=(RF*H2(L)+H2(J+1))/(RF+1.)
CP(L)=1.-((UX(L)**2+UY(L)**2+UZ(L)**2)/USQ
UABS(L)=SQRT(1.-CP(L))
RF=RF+1.
GO TO 330
320 RF=1.
L=L+1
K=J+1
STML(L)=STML(K)
XL(L)=XL(K)
YL(L)=YL(K)
ZL(L)=ZL(K)
UX(L)=UX(K)
UY(L)=UY(K)

```

```

UZ(L)=UZ(K)
GK1(L)=GK1(K)
GK2(L)=GK2(K)
H2(L)=H2(K)
CP(L)=CP(K)
UABS(L)=UABS(K)
NQUAD(L)=NQUAD(K)

330 CONTINUE
L=L+1
STML(L)=STML(JMAX)
XL(L)=XL(JMAX)
YL(L)=YL(JMAX)
ZL(L)=ZL(JMAX)
UX(L)=UX(JMAX)
UY(L)=UY(JMAX)
UZ(L)=UZ(JMAX)
GK1(L)=GK1(JMAX)
GK2(L)=GK2(JMAX)
H2(L)=H2(JMAX)
CP(L)=CP(JMAX)
UABS(L)=UABS(JMAX)
JMAX=L
NQUAD(JMAX)=NQUAD(JMAX-1)
NQUAD(JMIN)=NQUAD(JMIN+1)
WRITE(6,51)
51 FORMAT(4HO 1,6X,1HX,9X,1HY,
19X,1HZ,09X,2HUX,8X,2HUY,8X,2HVZ,09X,
22HCP, 8X,2HK1, 8X,2HK2, 8X,2HH2,8X,2HSL,8X,1HU,9X,1HP)
IF (AMACH .EQ. 0.) GOTO 1160
C *** COMPUTE COMPRESSIBILITY CORRECTION
DO 1150 J=JMIN,JMAX
USD = (UX(J)**2+UY(J)**2+UZ(J)**2)/USQ
USDR = USD
SM = AMACH**2
DO 1140 I=1,3
R = (1.+.2*SM*(1.-USDR))
IF (R .LT. .000001) R = .000001
USDB = USD/R**2.5
R = (1.+.2*SM*(1.-USDB))
IF (R .LT. .000001) R = .000001
USDC = USD/R**2.5
1140 USDR = (USDC*USDR-USDB**2)/(USDC+USDR-2.*USDB)
R = (1.+.2*SM*(1.-USDR))
IF (R .LT. .000001) R = .000001
R = R**1.25
UX(J) = UX(J)/R
UY(J) = UY(J)/R
UZ(J) = UZ(J)/R
UABS(J) = SORT(USDR)
1150 CP(J) = (R**2.8-1.)/(.7*SM)
1150 CONTINUE
K=0
DO 53 I=JMIN,JMAX
K=K+1
53 WRITE(6,60) K,XL(I),YL(I),ZL(I),UX(I),UY(I),UZ(I),CP(I),
1 GK1(I),GK2(I),H2(I),STML(I),UABS(I),NQUAD(I)
60 FORMAT(1X,I3,3F10.5,1X,3F10.5,1X,6F10.5,1E)
8 FORMAT(3F12.5)
WRITE(17) K, (XL(I),YL(I),ZL(I),NQUAD(I), I=JMIN,JMAX)
C   IWRITE .LE. 0      --    WRITE SL,U,H2,K2
C   IWRITE .GE. 2      --    WRITE X,Y,Z,CP
C   IWRITE .EQ. 1      --    WRITE SL,U,H2,K2 AND X,Y,Z,CP

```

```
IF (IWRITE.GT.1) GO TO 340
WRITE(7) K, (STML(I),URBS(I),H2(I),GK2(I), I=JMIN,JMAX)
340 IF (IWRITE.LT.1) GO TO 400
WRITE(7) K, (XL(I),YL(I),ZL(I),CP(I), I=JMIN,JMAX)
GO TO 400
300 WRITE(6,50) NSP(LL)
WRITE(6,65)
GO TO 282
400 CONTINUE
GO TO 100
C READ NEXT SET OF STREAMLINES
450 WRITE(6,451)IS,NQ
451 FORMAT(14H TAPE 04 ERROR,214)
550 ENDFILE 7
REWIND 7
ENDFILE 17
REWIND 17
REWIND 04
STOP ?
END
```

APPENDIX VIII - TRIAXIAL ELLIPSOID INPUT FILE

SAMPLE PROBLEM TRIAXIAL ELLIPSOID

280	5	150	150	150	3	.00001	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.	00000	.00000	.00000	1	1	1	0	.00000														
97861	.00000	.10286	1	2	1	0	.00000															
90789	.00000	.20960	1	3	1	0	.00000															
82360	.00000	.28359	1	4	1	0	.00000															
71583	.00000	.34914	1	5	1	0	.00000															
62478	.00000	.39040	1	6	1	0	.00000															
52537	.00000	.42544	1	7	1	0	.00000															
44721	.00000	.44721	1	8	1	0	.00000															
37530	.00000	.46345	1	9	1	0	.00000															
29822	.00000	.47725	1	10	1	0	.00000															
23692	.00000	.48576	1	11	1	0	.00000															
16957	.00000	.49275	1	12	1	0	.00000															
11457	.00000	.49670	1	13	1	0	.00000															
05248	.00000	.49931	1	14	1	0	.00000															
00000	.00000	.50000	1	15	1	0	.00000															
99875	.10000	.00000	2	1	1	0	.00000															
97739	.10000	.10273	2	2	1	0	.00000															
90676	.10000	.20934	2	3	1	0	.00000															
82257	.10000	.28323	2	4	1	0	.00000															
71494	.10000	.34870	2	5	1	0	.00000															
62399	.10000	.38991	2	6	1	0	.00000															
52471	.10000	.42490	2	7	1	0	.00000															
44565	.10000	.44665	2	8	1	0	.00000															
37485	.10000	.46267	2	9	1	0	.00000															
29785	.10000	.47665	2	10	1	0	.00000															
23663	.10000	.48516	2	11	1	0	.00000															
16946	.10000	.49213	2	12	1	0	.00000															
11453	.10000	.49608	2	13	1	0	.00000															
05241	.10000	.49859	2	14	1	0	.00000															
00000	.10000	.49937	2	15	1	0	.00000															
99499	.20000	.00000	3	1	1	0	.00000															
97371	.20000	.10234	3	2	1	0	.00000															
90334	.20000	.20855	3	3	1	0	.00000															
81947	.20000	.28217	3	4	1	0	.00000															
71225	.20000	.34739	3	5	1	0	.00000															
62164	.20000	.38845	3	6	1	0	.00000															
52274	.20000	.42330	3	7	1	0	.00000															
44497	.20000	.44497	3	8	1	0	.00000															
37342	.20000	.45113	3	9	1	0	.00000															
29672	.20000	.47486	3	10	1	0	.00000															
23574	.20000	.48333	3	11	1	0	.00000															
16992	.20000	.49028	3	12	1	0	.00000															
11410	.20000	.49421	3	13	1	0	.00000															
05222	.20000	.49581	3	14	1	0	.00000															
00000	.20000	.49749	3	15	1	0	.00000															
98869	.30000	.00000	4	1	1	0	.00000															
96754	.30000	.10169	4	2	1	0	.00000															
89762	.30000	.20723	4	3	1	0	.00000															
81428	.30000	.28038	4	4	1	0	.00000															
70774	.30000	.34519	4	5	1	0	.00000															
61771	.30000	.36599	4	6	1	0	.00000															
51943	.30000	.42062	4	7	1	0	.00000															
44215	.30000	.44215	4	8	1	0	.00000															
37105	.30000	.45821	4	9	1	0	.00000															
29464	.30000	.47185	4	10	1	0	.00000															
23424	.30000	.48027	4	11	1	0	.00000															
16775	.30000	.48718	4	12	1	0	.00000															

.11338	.30000	.49108	4	13	1	0	.00000
.05189	.30000	.49366	4	14	1	0	.00000
.00000	.30000	.49434	4	15	1	0	.00000
.97980	.40000	.00000	5	1	1	0	.00000
.95884	.40000	.10078	5	2	1	0	.00000
.88955	.40000	.20537	5	3	1	0	.00000
.80696	.40000	.27786	5	4	1	0	.00000
.70137	.40000	.34208	5	5	1	0	.00000
.61215	.40000	.39251	5	6	1	0	.00000
.51476	.40000	.41684	5	7	1	0	.00000
.43818	.40000	.43818	5	8	1	0	.00000
.36771	.40000	.45409	5	9	1	0	.00000
.29219	.40000	.46761	5	10	1	0	.00000
.23214	.40000	.47595	5	11	1	0	.00000
.16624	.40000	.48280	5	12	1	0	.00000
.11236	.40000	.48667	5	13	1	0	.00000
.05142	.40000	.49922	5	14	1	0	.00000
.00000	.40000	.48990	5	15	1	0	.00000
.97980	.40000	.00000	6	1	2	0	.00000
.95884	.40000	.10078	6	2	2	0	.00000
.88955	.40000	.20537	6	3	2	0	.00000
.80696	.40000	.27786	6	4	2	0	.00000
.70137	.40000	.34208	6	5	2	0	.00000
.61215	.40000	.39251	6	6	2	0	.00000
.51476	.40000	.41684	6	7	2	0	.00000
.43818	.40000	.43818	6	8	2	0	.00000
.36771	.40000	.45409	6	9	2	0	.00000
.29219	.40000	.46761	6	10	2	0	.00000
.23214	.40000	.47595	6	11	2	0	.00000
.16624	.40000	.48280	6	12	2	0	.00000
.11236	.40000	.48667	6	13	2	0	.00000
.05142	.40000	.49922	6	14	2	0	.00000
.00000	.40000	.48990	6	15	2	0	.00000
.96825	.50000	.00000	7	1	2	0	.00000
.94754	.50000	.09959	7	2	2	0	.00000
.87936	.50000	.20295	7	3	2	0	.00000
.79745	.50000	.27458	7	4	2	0	.00000
.69310	.50000	.33805	7	5	2	0	.00000
.60434	.50000	.37801	7	6	2	0	.00000
.50869	.50000	.41193	7	7	2	0	.00000
.43301	.50000	.43301	7	8	2	0	.00000
.35338	.50000	.44874	7	9	2	0	.00000
.26875	.50000	.46209	7	10	2	0	.00000
.22940	.50000	.47034	7	11	2	0	.00000
.16428	.50000	.47710	7	12	2	0	.00000
.11103	.50000	.49093	7	13	2	0	.00000
.05081	.50000	.48345	7	14	2	0	.00000
.00000	.50000	.48412	7	15	2	0	.00000
.95334	.60000	.00000	8	1	2	0	.00000
.93354	.60000	.09912	8	2	2	0	.00000
.85507	.60000	.19995	8	3	2	0	.00000
.78566	.60000	.27052	8	4	2	0	.00000
.69286	.60000	.33305	8	5	2	0	.00000
.59500	.60000	.37242	8	6	2	0	.00000
.50117	.60000	.40564	8	7	2	0	.00000
.42652	.60000	.42651	8	8	2	0	.00000
.35901	.60000	.44211	8	9	2	0	.00000
.28448	.60000	.45527	8	10	2	0	.00000
.22601	.60000	.46339	8	11	2	0	.00000
.16125	.60000	.47005	8	12	2	0	.00000
.10939	.60000	.47392	8	13	2	0	.00000

.05006	.60000	.47631	8	14	2	0	.00000
.00000	.60000	.47697	8	15	2	0	.00000
.93675	.70000	.00000	9	1	2	0	.00000
.91671	.70000	.09635	9	2	2	0	.00000
.85047	.70000	.19635	9	3	2	0	.00000
.77150	.70000	.25565	9	4	2	0	.00000
.67056	.70000	.32705	9	5	2	0	.00000
.58526	.70000	.36571	9	6	2	0	.00000
.49214	.70000	.39853	9	7	2	0	.00000
.41893	.70000	.41893	9	8	2	0	.00000
.35156	.70000	.43414	9	9	2	0	.00000
.27936	.70000	.44706	9	10	2	0	.00000
.22194	.70000	.45504	9	11	2	0	.00000
.15894	.70000	.45158	9	12	2	0	.00000
.10742	.70000	.46529	9	13	2	0	.00000
.04916	.70000	.46773	9	14	2	0	.00000
.00000	.70000	.46837	9	15	2	0	.00000
.91652	.80000	.00000	10	1	2	0	.00000
.89691	.80000	.09427	10	2	2	0	.00000
.83210	.80000	.19210	10	3	2	0	.00000
.75484	.80000	.25991	10	4	2	0	.00000
.65607	.80000	.31999	10	5	2	0	.00000
.57262	.80000	.35781	10	6	2	0	.00000
.48151	.80000	.38992	10	7	2	0	.00000
.40988	.80000	.40988	10	8	2	0	.00000
.34397	.80000	.42476	10	9	2	0	.00000
.27332	.80000	.43741	10	10	2	0	.00000
.21714	.80000	.44521	10	11	2	0	.00000
.15550	.80000	.45161	10	12	2	0	.00000
.10510	.80000	.45523	10	13	2	0	.00000
.04810	.80000	.45763	10	14	2	0	.00000
.00000	.80000	.45826	10	15	2	0	.00000
.91652	.80000	.00000	11	1	3	0	.00000
.89691	.80000	.09427	11	2	3	0	.00000
.83210	.80000	.19210	11	3	3	0	.00000
.75484	.80000	.25991	11	4	3	0	.00000
.65607	.80000	.31999	11	5	3	0	.00000
.57262	.80000	.35781	11	6	3	0	.00000
.48151	.80000	.38992	11	7	3	0	.00000
.40988	.80000	.40988	11	8	3	0	.00000
.34397	.80000	.42476	11	9	3	0	.00000
.27332	.80000	.43741	11	10	3	0	.00000
.21714	.80000	.44521	11	11	3	0	.00000
.15550	.80000	.45161	11	12	3	0	.00000
.10510	.80000	.45523	11	13	3	0	.00000
.04810	.80000	.45763	11	14	3	0	.00000
.00000	.80000	.45826	11	15	3	0	.00000
.89303	.90000	.00000	12	1	3	0	.00000
.87393	.90000	.09185	12	2	3	0	.00000
.81077	.90000	.18718	12	3	3	0	.00000
.73550	.90000	.25325	12	4	3	0	.00000
.63926	.90000	.31179	12	5	3	0	.00000
.55794	.90000	.34854	12	6	3	0	.00000
.46917	.90000	.37993	12	7	3	0	.00000
.39937	.90000	.39937	12	8	3	0	.00000
.33515	.90000	.41358	12	9	3	0	.00000
.26632	.90000	.42620	12	10	3	0	.00000
.21158	.90000	.43390	12	11	3	0	.00000
.15152	.90000	.44004	12	12	3	0	.00000
.10241	.90000	.44357	12	13	3	0	.00000
.04687	.90000	.44590	12	14	3	0	.00000

.00000	.90000	.44651	12	15	3	0	.00000
.86603	1.00000	.00000	13	1	3	0	.00000
.84750	1.00000	.06908	13	2	3	0	.00000
.78626	1.00000	.18152	13	3	3	0	.00000
.71326	1.00000	.24559	13	4	3	0	.00000
.61993	1.00000	.30236	13	5	3	0	.00000
.54107	1.00000	.33810	13	6	3	0	.00000
.45498	1.00000	.35844	13	7	3	0	.00000
.38730	1.00000	.38730	13	8	3	0	.00000
.32502	1.00000	.40136	13	9	3	0	.00000
.25826	1.00000	.41331	13	10	3	0	.00000
.20518	1.00000	.42068	13	11	3	0	.00000
.14694	1.00000	.42673	13	12	3	0	.00000
.09931	1.00000	.43016	13	13	3	0	.00000
.04545	1.00000	.43242	13	14	3	0	.00000
.00000	1.00000	.43301	13	15	3	0	.00000
.83516	1.10000	.00000	14	1	3	0	.00000
.81730	1.10000	.08590	14	2	3	0	.00000
.75624	1.10000	.17505	14	3	3	0	.00000
.68784	1.10000	.23684	14	4	3	0	.00000
.59784	1.10000	.29159	14	5	3	0	.00000
.52179	1.10000	.32605	14	6	3	0	.00000
.43877	1.10000	.35531	14	7	3	0	.00000
.37350	1.10000	.37350	14	8	3	0	.00000
.31343	1.10000	.38706	14	9	3	0	.00000
.24905	1.10000	.39858	14	10	3	0	.00000
.19787	1.10000	.40599	14	11	3	0	.00000
.14170	1.10000	.41153	14	12	3	0	.00000
.09577	1.10000	.41483	14	13	3	0	.00000
.04383	1.10000	.41701	14	14	3	0	.00000
.00000	1.10000	.41758	14	15	3	0	.00000
.80000	1.20000	.00000	15	1	3	0	.00000
.78269	1.20000	.08228	15	2	3	0	.00000
.72631	1.20000	.16768	15	3	3	0	.00000
.55863	1.20000	.22687	15	4	3	0	.00000
.57267	1.20000	.27931	15	5	3	0	.00000
.49982	1.20000	.31232	15	6	3	0	.00000
.42030	1.20000	.34035	15	7	3	0	.00000
.35777	1.20000	.35777	15	8	3	0	.00000
.30024	1.20000	.37076	15	9	3	0	.00000
.23957	1.20000	.38180	15	10	3	0	.00000
.18954	1.20000	.38861	15	11	3	0	.00000
.13573	1.20000	.39420	15	12	3	0	.00000
.09174	1.20000	.39736	15	13	3	0	.00000
.04198	1.20000	.39945	15	14	3	0	.00000
.00000	1.20000	.40000	15	15	3	0	.00000
.80000	1.20000	.00000	16	1	4	0	.00000
.78269	1.20000	.08228	16	2	4	0	.00000
.72631	1.20000	.16768	16	3	4	0	.00000
.55863	1.20000	.22687	16	4	4	0	.00000
.57267	1.20000	.27931	16	5	4	0	.00000
.49982	1.20000	.31232	16	6	4	0	.00000
.42030	1.20000	.34035	16	7	4	0	.00000
.35777	1.20000	.35777	16	8	4	0	.00000
.30024	1.20000	.37076	16	9	4	0	.00000
.23957	1.20000	.38180	16	10	4	0	.00000
.18954	1.20000	.38861	16	11	4	0	.00000
.13573	1.20000	.39420	16	12	4	0	.00000
.09174	1.20000	.39736	16	13	4	0	.00000
.04198	1.20000	.39945	16	14	4	0	.00000
.00000	1.20000	.40000	16	15	4	0	.00000

75993	1.30000	00000	17	1	4	0	.00000
74358	1.30000	07816	17	2	4	0	.00000
68994	1.30000	15928	17	3	4	0	.00000
62588	1.30000	21551	17	4	4	0	.00000
54399	1.30000	26532	17	5	4	0	.00000
47479	1.30000	29668	17	6	4	0	.00000
39925	1.30000	32330	17	7	4	0	.00000
33985	1.30000	33985	17	8	4	0	.00000
29520	1.30000	35219	17	9	4	0	.00000
22663	1.30000	36268	17	10	4	0	.00000
18005	1.30000	36915	17	11	4	0	.00000
12894	1.30000	37446	17	12	4	0	.00000
08714	1.30000	37746	17	13	4	0	.00000
03988	1.30000	37944	17	14	4	0	.00000
00000	1.30000	37997	17	15	4	0	.00000
71414	1.40000	00000	18	1	4	0	.00000
69887	1.40000	07345	18	2	4	0	.00000
64836	1.40000	14969	18	3	4	0	.00000
58817	1.40000	20252	18	4	4	0	.00000
51121	1.40000	24933	18	5	4	0	.00000
44513	1.40000	27880	18	6	4	0	.00000
37519	1.40000	30382	18	7	4	0	.00000
31937	1.40000	31937	18	8	4	0	.00000
26922	1.40000	33097	18	9	4	0	.00000
21297	1.40000	34082	18	10	4	0	.00000
16920	1.40000	34690	18	11	4	0	.00000
12117	1.40000	35189	18	12	4	0	.00000
08189	1.40000	35472	18	13	4	0	.00000
03748	1.40000	35558	18	14	4	0	.00000
00000	1.40000	35707	18	15	4	0	.00000
66144	1.50000	00000	19	1	4	0	.00000
64729	1.50000	06803	19	2	4	0	.00000
63051	1.50000	13854	19	3	4	0	.00000
54476	1.50000	18758	19	4	4	0	.00000
47348	1.50000	23093	19	5	4	0	.00000
41325	1.50000	25823	19	6	4	0	.00000
34750	1.50000	28140	19	7	4	0	.00000
29580	1.50000	29580	19	8	4	0	.00000
24824	1.50000	30654	19	9	4	0	.00000
19725	1.50000	31567	19	10	4	0	.00000
15671	1.50000	32130	19	11	4	0	.00000
11222	1.50000	32592	19	12	4	0	.00000
07585	1.50000	32854	19	13	4	0	.00000
03471	1.50000	33026	19	14	4	0	.00000
00000	1.50000	33072	19	15	4	0	.00000
63020	1.60000	00000	20	1	4	0	.00000
58717	1.60000	06171	20	2	4	0	.00000
54473	1.60000	12576	20	3	4	0	.00000
49416	1.60000	17015	20	4	4	0	.00000
42950	1.60000	20948	20	5	4	0	.00000
37487	1.60000	23424	20	6	4	0	.00000
31522	1.60000	25526	20	7	4	0	.00000
26223	1.60000	26823	20	8	4	0	.00000
22518	1.60000	27807	20	9	4	0	.00000
17893	1.60000	28635	20	10	4	0	.00000
14215	1.60000	29145	20	11	4	0	.00000
10180	1.60000	29565	20	12	4	0	.00000
06880	1.60000	29802	20	13	4	0	.00000
03149	1.60000	29959	20	14	4	0	.00000
00000	1.60000	30000	20	15	4	0	.00000
00000	1.60000	00000	21	1	5	0	.00000

.58717	1.60000	.06171	21	2	5	0	.00000
.54473	1.60000	.12576	21	3	5	0	.00000
.49416	1.60000	.17015	21	4	5	0	.00000
.42950	1.60000	.20948	21	5	5	0	.00000
.37487	1.60000	.23424	21	6	5	0	.00000
.31522	1.60000	.25526	21	7	5	0	.00000
.26833	1.60000	.26833	21	8	5	0	.00000
.22518	1.60000	.27807	21	9	5	0	.00000
.17893	1.60000	.28635	21	10	5	0	.00000
.14215	1.60000	.29146	21	11	5	0	.00000
.10180	1.60000	.29565	21	12	5	0	.00000
.06880	1.60000	.29802	21	13	5	0	.00000
.03149	1.60000	.29959	21	14	5	0	.00000
.00000	1.60000	.30000	21	15	5	0	.00000
.52678	1.70000	.00000	22	1	5	0	.00000
.51552	1.70000	.05418	22	2	5	0	.00000
.47826	1.70000	.11042	22	3	5	0	.00000
.43386	1.70000	.14939	22	4	5	0	.00000
.37709	1.70000	.18392	22	5	5	0	.00000
.32912	1.70000	.20566	22	6	5	0	.00000
.27675	1.70000	.22411	22	7	5	0	.00000
.23558	1.70000	.23558	22	8	5	0	.00000
.19770	1.70000	.24414	22	9	5	0	.00000
.15710	1.70000	.25141	22	10	5	0	.00000
.12481	1.70000	.25589	22	11	5	0	.00000
.08938	1.70000	.25957	22	12	5	0	.00000
.05041	1.70000	.26165	22	13	5	0	.00000
.02765	1.70000	.26303	22	14	5	0	.00000
.00000	1.70000	.26339	22	15	5	0	.00000
.43539	1.80000	.00000	23	1	5	0	.00000
.42657	1.80000	.04483	23	2	5	0	.00000
.39574	1.80000	.09135	23	3	5	0	.00000
.35900	1.80000	.12361	23	4	5	0	.00000
.31202	1.80000	.15218	23	5	5	0	.00000
.27233	1.80000	.17017	23	6	5	0	.00000
.22900	1.80000	.18544	23	7	5	0	.00000
.19494	1.80000	.19494	23	8	5	0	.00000
.16359	1.80000	.20201	23	9	5	0	.00000
.12999	1.80000	.20803	23	10	5	0	.00000
.10327	1.80000	.21174	23	11	5	0	.00000
.07395	1.80000	.21479	23	12	5	0	.00000
.04995	1.80000	.21651	23	13	5	0	.00000
.02288	1.80000	.21764	23	14	5	0	.00000
.00000	1.80000	.21794	23	15	5	0	.00000
.31225	1.90000	.00000	24	1	5	0	.00000
.32587	1.90000	.03212	24	2	5	0	.00000
.28349	1.90000	.06545	24	3	5	0	.00000
.25717	1.90000	.08855	24	4	5	0	.00000
.22352	1.90000	.10902	24	5	5	0	.00000
.19509	1.90000	.12190	24	6	5	0	.00000
.16405	1.90000	.13234	24	7	5	0	.00000
.13964	1.90000	.13954	24	8	5	0	.00000
.11719	1.90000	.14471	24	9	5	0	.00000
.09312	1.90000	.14932	24	10	5	0	.00000
.07356	1.90000	.15168	24	11	5	0	.00000
.05292	1.90000	.15295	24	12	5	0	.00000
.03581	1.90000	.15510	24	13	5	0	.00000
.01629	1.90000	.15591	24	14	5	0	.00000
.00000	1.90000	.15612	24	15	5	0	.00000
.00000	2.00000	.06200	25	1	5	0	.00000
.00000	2.00000	.00000	25	2	5	0	.00000

.00000	2.00000	.00000	25	3	5	0	.00000
.00000	2.00000	.00000	25	4	5	0	.00000
.00000	2.00000	.00000	25	5	5	0	.00000
.00000	2.00000	.00000	25	6	5	0	.00000
.00000	2.00000	.00000	25	7	5	0	.00000
.00000	2.00000	.00000	25	8	5	0	.00000
.00000	2.00000	.00000	25	9	5	0	.00000
.00000	2.00000	.00000	25	10	5	0	.00000
.00000	2.00000	.00000	25	11	5	0	.00000
.00000	2.00000	.00000	25	12	5	0	.00000
.00000	2.00000	.00000	25	13	5	0	.00000
.00000	2.00003	.00000	25	14	5	0	.00000
.00000	2.00000	.00000	25	15	5	0	.00000

(EOR)

3	0	0				
2.00000	.00000	.00000				
.00000	.00000	1.50000				
.00000	3.00000	.00000				

(EDR)

2	20	1	-1.00000	.10000	.00000	.00000
1.00000			1.00000	.00000		
1.50000			.66666	.00000		

(EOR)

-1.0000	.0000	.0000	1	0	1	
1.0000	.0500	.0000		1		

(EOF)

(EOF)

APPENDIX IX - MAXIMIZING ELLIPTOID INPUT FILE

XZY POTENTIAL FLUX PROJECTION SECTION TO VTKLJL, 4
SAMPLE PROBLEM TRIAXIAL ELLIPSOID

NU. OF QUADS = 280
NU. OF SECTIONS = 5
MAX. NO. OF ITERATIONS = FLUX 150 + FLUX 150 + FLUX 150
3 PLANES OF SYMMETRY
CONVERGENCE CRITERIA = 0.0001

ISPF = 0

IEDIT1 = 0
IEDIT3 = 0
IEDIT4 = 0
ITAPE = 0

XCENTER = .00
YCENTER = .00
ZCENTER = .00

7	• 52471t+0j	• 44667t+0j	• 2227t+0j	• 44477t+0j	• 26852t+0j	• 89900t+0j	• 32183t+0j
2	• 10207t+0j	• 103t+0j	• 270t+0j	• 1447t+0j	• 20747t+0j	• 64733t+0j	• 66965t+0j
4	• 42450t+0j	• 44665t+0j	• 4230t+0j	• 4546t+0j	• 40310t+0j	• 64804t+0j	• 85764t+0j
4	• 44721t+0j	• 37539t+0j	• 37463t+0j	• 44666t+0j	• 4410t+0j	• 22024t+0j	• 32082t+0j
6	• 00000t+0j	• 00000t+0j	• 10000t+0j	• 10000t+0j	• 46605t+0j	• 6703t+0j	• 67036t+0j
5	• 44721t+0j	• 46345t+0j	• 42627t+0j	• 44665t+0j	• 45505t+0j	• 97541t+0j	• 24607t+0j
8	• 44665t+0j	• 37463t+0j	• 44477t+0j	• 40947t+0j	• 22026t+0j	• 73505t+0j	• 67020t+0j
2	• 10000t+0j	• 10000t+0j	• 20000t+0j	• 19947t+0j	• 20108t+0j	• 32108t+0j	• 32108t+0j
6	• 44665t+0j	• 37463t+0j	• 44477t+0j	• 45394t+0j	• 97523t+0j	• 60228t+0j	• 94007t+0j
9	• 37539t+0j	• 24705t+0j	• 24705t+0j	• 33655t+0j	• 17622t+0j	• 78256t+0j	• 65349t+0j
1	• 00000t+0j	• 00000t+0j	• 10000t+0j	• 99433t+0j	• 65574t+0j	• 63586t+0j	• 27940t+0j
7	• 46345t+0j	• 47745t+0j	• 47627t+0j	• 40267t+0j	• 47008t+0j	• 59108t+0j	• 35907t+0j
9	• 37483t+0j	• 24705t+0j	• 246772t+0j	• 35571t+0j	• 17617t+0j	• 78076t+0j	• 27971t+0j
2	• 10000t+0j	• 10000t+0j	• 20000t+0j	• 14447t+0j	• 17635t+0j	• 63728t+0j	• 65426t+0j
P	• 46267t+0j	• 47605t+0j	• 47605t+0j	• 4660t+0j	• 98416t+0j	• 58967t+0j	• 52681t+0j
0	• 29822t+0j	• 236492t+0j	• 23663t+0j	• 26741t+0j	• 13754t+0j	• 61850t+0j	• 27942t+0j
1	• 00000t+0j	• 00000t+0j	• 10000t+0j	• 49484t+0j	• 64023t+0j	• 50826t+0j	• 64671t+0j
9	• 47725t+0j	• 48556t+0j	• 48556t+0j	• 48121t+0j	• 94047t+0j	• 42746t+0j	• 31447t+0j
0	• 297885t+0j	• 23663t+0j	• 23574t+0j	• 26674t+0j	• 13760t+0j	• 61699t+0j	• 64660t+0j
2	• 10000t+0j	• 10000t+0j	• 20000t+0j	• 14497t+0j	• 19351t+0j	• 58954t+0j	• 27975t+0j
0	• 47605t+0j	• 48556t+0j	• 48533t+0j	• 48000t+0j	• 94030t+0j	• 42729t+0j	• 61924t+0j
1	• 236492t+0j	• 16967t+0j	• 16967t+0j	• 23633t+0j	• 10330t+0j	• 67573t+0j	• 68855t+0j
1	• 00000t+0j	• 00000t+0j	• 10000t+0j	• 49440t+0j	• 63343t+0j	• 60397t+0j	• 25888t+0j
1	• 49476t+0j	• 49476t+0j	• 494213t+0j	• 49516t+0j	• 90845t+0j	• 45363t+0j	• 13246t+0j
1	• 236492t+0j	• 169491t+0j	• 169491t+0j	• 23633t+0j	• 20317t+0j	• 67518t+0j	• 25913t+0j
2	• 10000t+0j	• 10000t+0j	• 20000t+0j	• 14497t+0j	• 19106t+0j	• 64660t+0j	• 63991t+0j
2	• 4516t+0j	• 4516t+0j	• 45213t+0j	• 45333t+0j	• 4673t+0j	• 45363t+0j	• 36368t+0j
1	• 236492t+0j	• 169491t+0j	• 169491t+0j	• 23633t+0j	• 20266t+0j	• 67418t+0j	• 25885t+0j
2	• 10000t+0j	• 10000t+0j	• 20000t+0j	• 14497t+0j	• 19351t+0j	• 60397t+0j	• 25888t+0j
2	• 4516t+0j	• 4516t+0j	• 45213t+0j	• 45333t+0j	• 4673t+0j	• 45363t+0j	• 36368t+0j
1	• 236492t+0j	• 169491t+0j	• 169491t+0j	• 23633t+0j	• 20317t+0j	• 67518t+0j	• 25913t+0j
2	• 10000t+0j	• 10000t+0j	• 20000t+0j	• 14497t+0j	• 19351t+0j	• 60397t+0j	• 25888t+0j
2	• 4516t+0j	• 4516t+0j	• 45213t+0j	• 45333t+0j	• 4673t+0j	• 45363t+0j	• 36368t+0j
1	• 236492t+0j	• 169491t+0j	• 169491t+0j	• 23633t+0j	• 20317t+0j	• 67518t+0j	• 25913t+0j
3	• 45275t+0j	• 45275t+0j	• 45333t+0j	• 45442t+0j	• 99741t+0j	• 35406t+0j	• 15840t+0j
3	• 16967t+0j	• 11493t+0j	• 11493t+0j	• 16946t+0j	• 71667t+0j	• 54976t+0j	• 63170t+0j
2	• 10000t+0j	• 10000t+0j	• 20000t+0j	• 16943t+0j	• 57160t+0j	• 57160t+0j	• 25915t+0j
2	• 45275t+0j	• 45275t+0j	• 45333t+0j	• 45442t+0j	• 99741t+0j	• 35406t+0j	• 15840t+0j
4	• 45275t+0j	• 45275t+0j	• 45333t+0j	• 45442t+0j	• 99741t+0j	• 35406t+0j	• 15840t+0j
3	• 11493t+0j	• 52470t+0j	• 52470t+0j	• 11493t+0j	• 41954t+0j	• 62211t+0j	• 63181t+0j
1	• 00000t+0j	• 00000t+0j	• 10000t+0j	• 45442t+0j	• 52470t+0j	• 52422t+0j	• 63182t+0j
5	• 45275t+0j	• 45275t+0j	• 45333t+0j	• 45442t+0j	• 99910t+0j	• 40223t+0j	• 70339t+0j
3	• 11493t+0j	• 52470t+0j	• 52470t+0j	• 11493t+0j	• 41954t+0j	• 62211t+0j	• 63181t+0j
2	• 10000t+0j	• 10000t+0j	• 20000t+0j	• 45442t+0j	• 52470t+0j	• 52422t+0j	• 63182t+0j
6	• 45275t+0j	• 45275t+0j	• 45333t+0j	• 45442t+0j	• 99910t+0j	• 40223t+0j	• 70339t+0j

N	Y ₁	Y ₂	Y ₃	Y ₄	Y ₅	Y ₆	Y ₇	Y ₈	Y ₉	Y ₁₀	Y ₁₁	Y ₁₂	Y ₁₃	Y ₁₄
P	Z ₁	Z ₂	Z ₃	Z ₄	Z ₅	Z ₆	Z ₇	Z ₈	Z ₉	Z ₁₀	Z ₁₁	Z ₁₂	Z ₁₃	Z ₁₄
13	.11416t+00	.5222e-01	.5222e-01	.5222e-01	.5222e-01	.5222e-01								
3	.26000t+00	.2090e-00	.2090e-00	.2090e-00	.2090e-00	.2090e-00								
53	.49421t+00	.49661t+00	.49661t+00	.49661t+00	.49661t+00	.49661t+00								
13	.11338t+00	.51670t-01	.51670t-01	.51670t-01	.51670t-01	.51670t-01								
4	.30000t+00	.30000t+00	.30000t+00	.30000t+00	.30000t+00									
54	.44108t+00	.44306t+00	.44306t+00	.44306t+00	.44306t+00	.44306t+00								
14	.52220t-01	.70003t+00	.70003t+00	.70003t+00	.70003t+00	.70003t+00								
3	.20050t+00	.26000t+00	.26000t+00	.26000t+00	.26000t+00	.26000t+00								
55	.49681t+00	.49749t+00	.49749t+00	.49749t+00	.49749t+00	.49749t+00								
14	.51840t-01	.00006t+00	.00006t+00	.00006t+00	.00006t+00	.00006t+00								
4	.36000t+00	.33000t+00	.33000t+00	.33000t+00	.33000t+00	.33000t+00								
56	.49306t+00	.49439t+00	.49439t+00	.49439t+00	.49439t+00	.49439t+00								

		X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15	X16	X17	X18	X19	X20	X21	X22	X23	X24	X25	X26	X27	X28	X29	X30
		Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9	Y10	Y11	Y12	Y13	Y14	Y15	Y16	Y17	Y18	Y19	Y20	Y21	Y22	Y23	Y24	Y25	Y26	Y27	Y28	Y29	Y30
		Z1	Z2	Z3	Z4	Z5	Z6	Z7	Z8	Z9	Z10	Z11	Z12	Z13	Z14	Z15	Z16	Z17	Z18	Z19	Z20	Z21	Z22	Z23	Z24	Z25	Z26	Z27	Z28	Z29	Z30
P																															
13	.10939E+00	.50060E-01	.49160E-01	.10742E+00	.74010E-01	.41734E-01	.54004E-02	.-75765E-01																							
8	.60000E+00	.60000E+00	.70000E+00	.70000E+00	.64962E+00	.64962E+00	.56448E-01	.-26408E+00																							
109	.47382E+00	.47631E+00	.47631E+00	.46529E+00	.46529E+00	.47000E+00	.49544E+00	.41959E-03	.27733E-02																						
13	.10742E+00	.49160E-01	.46100E-01	.10210E+00	.77448E-01	.41741E-01	.57977E-02	.-27066E+00																							
9	.70000E+00	.70000E+00	.80000E+00	.80000E+00	.74962E+00	.74962E+00	.10047E+00	.-75826E-01																							
110	.46529E+00	.46773E+00	.46773E+00	.45523E+00	.45523E+00	.46149E+00	.49440E+00	.41551E-03	.90339E-02																						
14	.50060E-01	.00060E+00	.00060E+00	.49160E-01	.24806E-01	.13052E-01	.49797E-02	.-26466E+00																							
8	.60000E+00	.60000E+00	.70000E+00	.70000E+00	.64962E+00	.64962E+00	.85672E-01	.-74860E-01																							
111	.47631E+00	.47647E+00	.47647E+00	.45537E+00	.45537E+00	.47236E+00	.99624E+00	.35211E-03	.-14001E-03																						
14	.49160E-01	.00060E+00	.00060E+00	.48160E-01	.24316E-01	.12991E-01	.46862E-02	.-74861E-01																							
9	.70000E+00	.70000E+00	.80000E+00	.80000E+00	.74962E+00	.74962E+00	.10064E+00	.-27066E+00																							
112	.46773E+00	.46837E+00	.46837E+00	.45763E+00	.45763E+00	.46302E+00	.99464E+00	.34946E-03	.-69016E-03																						

\mathbf{M}	\mathbf{Y}_1	\mathbf{X}_2	\mathbf{A}_1	\mathbf{C}_{24}
\mathbf{N}	\mathbf{Y}_1	\mathbf{Y}_2	\mathbf{Y}_3	\mathbf{C}_5
\mathbf{P}	\mathbf{Z}_1	\mathbf{Z}_2	\mathbf{Z}_3	\mathbf{C}_6
1	$91422t+00$	$836911t+00$	$836911t+00$	$836911t+00$
11	$80000t+00$	$80000t+00$	$80000t+00$	$80000t+00$
13	$80000t+00$	$80000t+00$	$80000t+00$	$80000t+00$
1	$89303t+00$	$81343t+00$	$82603t+00$	$837049t+00$
12	$90000t+00$	$90000t+00$	$100000t+01$	$94974t+00$
14	$90000t+00$	$91456t+01$	$900000t+00$	$40532t-01$
1	$84691t+00$	$63210t+00$	$61077t+00$	$85347t+00$
11	$80000t+00$	$80000t+00$	$90000t+00$	$84678t+00$
15	$94270t-01$	$19240t+00$	$10718t+00$	$14136t+00$
2	$87393t+00$	$81077t+00$	$76626t+00$	$82568t+00$
12	$90000t+00$	$90000t+00$	$100000t+01$	$94474t+00$
16	$91850t-01$	$15718t+00$	$890000t-01$	$13742t+00$
2	$83210t+00$	$754t+00$	$71550t+00$	$76335t+00$
11	$80000t+00$	$90000t+00$	$90000t+00$	$84478t+00$
17	$19210t+00$	$25749t+00$	$22325t+00$	$18718t+00$
3	$81077t+00$	$73505t+00$	$71326t+00$	$70151t+00$
12	$90000t+00$	$90000t+00$	$100000t+01$	$94974t+00$
18	$18718t+00$	$25325t+00$	$18122t+00$	$21640t+00$
4	$75484t+00$	$63667t+00$	$63266t+00$	$73520t+00$
11	$80000t+00$	$80000t+00$	$90000t+00$	$84978t+00$
19	$25291t+00$	$31044t+00$	$31179t+00$	$23232t+00$
4	$73550t+00$	$63196t+00$	$61593t+00$	$71326t+00$
12	$90030t+00$	$90000t+00$	$100000t+01$	$94774t+00$
20	$25325t+00$	$31179t+00$	$31226t+00$	$24559t+00$
5	$65607t+00$	$57262t+00$	$57794t+00$	$60626t+00$
11	$80000t+00$	$90160t+00$	$94000t+00$	$84978t+00$
21	$31999t+00$	$35710t+00$	$34564t+00$	$31119t+00$
5	$63926t+00$	$55744t+00$	$51118t+00$	$58460t+00$
12	$90000t+00$	$90000t+00$	$100000t+01$	$94974t+00$
22	$31179t+00$	$34164t+00$	$33610t+00$	$32336t+00$
6	$57262t+00$	$48151t+00$	$40917t+00$	$557744t+00$
11	$80000t+00$	$80000t+00$	$90000t+00$	$90000t+00$
23	$35710t+00$	$35942t+00$	$35793t+00$	$34864t+00$
6	$55744t+00$	$46917t+00$	$44478t+00$	$54107t+00$
12	$90000t+00$	$90000t+00$	$100000t+01$	$94974t+00$
24	$34864t+00$	$37793t+00$	$35942t+00$	$35810t+00$
7	$46151t+00$	$44020t+00$	$35793t+00$	$46917t+00$
11	$80000t+00$	$80000t+00$	$90000t+00$	$94978t+00$
25	$36942t+00$	$38942t+00$	$37793t+00$	$35810t+00$

$\frac{N}{P}$	$\frac{Y_1}{Z_1}$	$\frac{Y_2}{Z_2}$	$\frac{Y_3}{Z_3}$	$\frac{Y_4}{Z_4}$	$\frac{Y_5}{Z_5}$	$\frac{Y_6}{Z_6}$	$\frac{Y_7}{Z_7}$	$\frac{Y_8}{Z_8}$	$\frac{Y_9}{Z_9}$	$\frac{Y_{10}}{Z_{10}}$	$\frac{Y_{11}}{Z_{11}}$	$\frac{Y_{12}}{Z_{12}}$	$\frac{Y_{13}}{Z_{13}}$	$\frac{Y_{14}}{Z_{14}}$	$\frac{Y_{15}}{Z_{15}}$	$\frac{Y_{16}}{Z_{16}}$	$\frac{Y_{17}}{Z_{17}}$	$\frac{Y_{18}}{Z_{18}}$	$\frac{Y_{19}}{Z_{19}}$	$\frac{Y_{20}}{Z_{20}}$
7	$46917L+00$	$29457L+00$	$30357L+00$	$30357L+00$	$4545L+00$	$4545L+00$	$4545L+00$	$37245L+00$	$37245L+00$	$26544L+00$	$77474L+00$									
12	$90000L+00$	$90000L+00$	$90000L+00$	$90000L+00$	$10000L+00$	$10000L+00$	$10000L+00$	$94474L+00$	$94474L+00$	$14733L+00$	$4474L+00$									
126	$37093L+00$	$37093L+00$	$37093L+00$	$37093L+00$	$30544L+00$	$30544L+00$	$30544L+00$	$30544L+00$	$30544L+00$	$45200L+00$										
8	$40988L+00$	$3437L+00$	$3437L+00$	$3437L+00$	$30357L+00$	$30357L+00$	$30357L+00$	$37245L+00$	$37245L+00$	$2185bt+00$	$37245L+00$									
11	$80000L+00$	$80000L+00$	$80000L+00$	$80000L+00$	$90000L+00$	$90000L+00$	$90000L+00$	$90000L+00$	$90000L+00$	$14000L+00$	$84474L+00$									
127	$40988L+00$	$42476L+00$	$42476L+00$	$42476L+00$	$41305L+00$	$41305L+00$	$41305L+00$	$3957L+00$	$3957L+00$	$41200L+00$										
8	$34957L+00$	$33515L+00$	$33515L+00$	$33515L+00$	$30357L+00$	$30357L+00$	$30357L+00$	$30357L+00$	$30357L+00$	$30357L+00$	$30357L+00$	$30357L+00$	$30357L+00$	$30357L+00$	$30357L+00$	$30357L+00$	$30357L+00$	$30357L+00$	$30357L+00$	
12	$90000L+00$	$90000L+00$	$90000L+00$	$90000L+00$	$10000L+00$	$10000L+00$	$10000L+00$	$10000L+00$	$10000L+00$	$10000L+00$	$10000L+00$	$10000L+00$	$10000L+00$	$10000L+00$	$10000L+00$	$10000L+00$	$10000L+00$	$10000L+00$	$10000L+00$	$10000L+00$
128	$34957L+00$	$41305L+00$	$41305L+00$	$41305L+00$	$40136L+00$	$40136L+00$	$40136L+00$	$367L+00$	$367L+00$	$40051L+00$										
9	$34347L+00$	$2732L+00$	$2732L+00$	$2732L+00$	$22066L+00$	$22066L+00$	$22066L+00$	$2036L+00$	$2036L+00$	$24621L+00$	$2036L+00$	$17441L+00$								
11	$80000L+00$	$80000L+00$	$80000L+00$	$80000L+00$	$90000L+00$	$90000L+00$	$90000L+00$	$90000L+00$	$90000L+00$	$94974L+00$	$94974L+00$	$12191L+00$								
129	$42476L+00$	$43741L+00$	$43741L+00$	$43741L+00$	$42620L+00$	$42620L+00$	$42620L+00$	$4136L+00$	$4136L+00$	$4136L+00$	$4136L+00$	$4136L+00$	$4136L+00$	$4136L+00$	$4136L+00$	$4136L+00$	$4136L+00$	$4136L+00$	$4136L+00$	
10	$27332L+00$	$21714L+00$	$21714L+00$	$21714L+00$	$21118L+00$	$21118L+00$	$21118L+00$	$20652L+00$	$20652L+00$	$24210L+00$	$20652L+00$	$13653L+00$								
11	$80000L+00$	$80000L+00$	$80000L+00$	$80000L+00$	$90000L+00$	$90000L+00$	$90000L+00$	$90000L+00$	$90000L+00$	$94974L+00$	$94974L+00$	$11908L+00$								
131	$43741L+00$	$44524L+00$	$44524L+00$	$44524L+00$	$43330L+00$	$43330L+00$	$43330L+00$	$426L+00$	$426L+00$	$43508L+00$	$426L+00$	$96336L+00$								
10	$26632L+00$	$2115L+00$	$2115L+00$	$2115L+00$	$20518L+00$	$20518L+00$	$20518L+00$	$20518L+00$	$20518L+00$	$23535L+00$	$20518L+00$	$13622L+00$								
12	$90000L+00$	$90000L+00$	$90000L+00$	$90000L+00$	$10000L+00$	$10000L+00$	$10000L+00$	$10000L+00$	$10000L+00$	$94974L+00$	$94974L+00$	$13754L+00$								
132	$42620L+00$	$43330L+00$	$43330L+00$	$43330L+00$	$42060L+00$	$42060L+00$	$42060L+00$	$41331L+00$	$41331L+00$	$42353L+00$	$41331L+00$	$96109L+00$								
11	$21714L+00$	$15550L+00$	$15550L+00$	$15550L+00$	$11112L+00$	$11112L+00$	$11112L+00$	$21118L+00$	$21118L+00$	$18395L+00$	$21118L+00$	$10237E+00$								
11	$80000L+00$	$80000L+00$	$80000L+00$	$80000L+00$	$90000L+00$	$90000L+00$	$90000L+00$	$90000L+00$	$90000L+00$	$84478L+00$	$84478L+00$	$11847L+00$								
133	$44521L+00$	$45111L+00$	$45111L+00$	$45111L+00$	$44004L+00$	$44004L+00$	$44004L+00$	$426L+00$	$426L+00$	$44269L+00$	$426L+00$	$96764L+00$								
11	$2115L+00$	$15152L+00$	$15152L+00$	$15152L+00$	$11461L+00$	$11461L+00$	$11461L+00$	$20518L+00$	$20518L+00$	$17802L+00$	$20518L+00$	$10237E+00$								
12	$90000L+00$	$94975L+00$	$94975L+00$	$11748L+00$																
134	$43330L+00$	$44004L+00$	$44004L+00$	$44004L+00$	$43330L+00$	$43330L+00$	$43330L+00$	$426L+00$	$426L+00$	$44303L+00$	$426L+00$	$94975L+00$								
11	$15550L+00$	$10540L+00$	$10540L+00$	$10540L+00$	$90321L+00$	$90321L+00$	$90321L+00$	$14644L+00$	$14644L+00$	$12055L+00$	$14644L+00$	$71115L+00$								
11	$80000L+00$	$80000L+00$	$80000L+00$	$80000L+00$	$90000L+00$	$90000L+00$	$90000L+00$	$90000L+00$	$90000L+00$	$94975L+00$	$94975L+00$	$11403L+00$								
135	$45111L+00$	$45523L+00$	$45523L+00$	$45523L+00$	$4473L+00$	$4473L+00$	$4473L+00$	$4457L+00$	$4457L+00$	$4457L+00$	$4457L+00$	$4457L+00$	$4457L+00$	$4457L+00$	$4457L+00$	$4457L+00$	$4457L+00$	$4457L+00$	$4457L+00$	
12	$15152L+00$	$10540L+00$	$10540L+00$	$10540L+00$	$90321L+00$	$90321L+00$	$90321L+00$	$14644L+00$	$14644L+00$	$12055L+00$	$14644L+00$	$71115L+00$								
136	$44004L+00$	4																		

7	• 39225t+00	• 41735t+00	• 37517t+00	• 26021t+00	• 61677t-02	• -42229t+00
7	• 12060t+01	• 13005t+01	• 14000t+01	• 15005t+01	• 16004t-01	• -15526t+00
7	• 32330t+00	• 35777t+00	• 39465t+00	• 42149t+00	• 45537t+00	• -13147t+00
8	• 35777t+00	• 30004t+01	• 26045t+00	• 26094t+00	• 26807t-02	• -40140t+00
8	• 12060t+01	• 12606t+01	• 13005t+01	• 13606t+01	• 14269t+01	• 24460t+00
8	• 35777t+00	• 37070t+00	• 39465t+00	• 39522t+00	• 40570t+00	• 76507t-03
9	• 39465t+00	• 26052t+00	• 31957t+00	• 36321t+00	• 41397t+00	• 58040t-02
9	• 13005t+01	• 13900t+01	• 14000t+01	• 14500t+01	• 15495t+01	• 20496t+00
9	• 35777t+00	• 35249t+00	• 35307t+00	• 35377t+00	• 35537t+00	• 70200t-03
9	• 30024t+00	• 23387t+00	• 26630t+00	• 26727t+00	• 27251t+00	• 55940t-02
9	• 12060t+01	• 12010t+01	• 12050t+01	• 12090t+01	• 12495t+01	• 20496t+00
9	• 35707t+00	• 35180t+00	• 35266t+00	• 35219t+00	• 35694t+00	• 69128t-03
9	• 28520t+00	• 22663t+00	• 21247t+00	• 20882t+00	• 17137t+00	• 54341t-02
9	• 13000t+01	• 13000t+01	• 14000t+01	• 14000t+01	• 15495t+01	• 61940t-01
9	• 35219t+00	• 36263t+00	• 34052t+00	• 35077t+00	• 34675t+00	• 67611t-03
0	• 23855t+00	• 18954t+00	• 16667t+00	• 20674t+00	• 13475t+00	• 49275t-02
0	• 12060t+01	• 12000t+01	• 12000t+01	• 12496t+01	• 20107t+00	• 50277t-01
0	• 38160t+00	• 36811t+00	• 35915t+00	• 36263t+00	• 37564t+00	• 97014t+00
0	• 22663t+00	• 18005t+00	• 16920t+00	• 21247t+00	• 19726t+00	• 46854t-02
0	• 13000t+01	• 13000t+01	• 14000t+01	• 14000t+01	• 15495t+01	• 58119t-01
0	• 36263t+00	• 36915t+00	• 34549t+00	• 34062t+00	• 35500t+00	• 90414t+00
1	• 18954t+00	• 13573t+00	• 12894t+00	• 16920t+00	• 19860t+00	• 53422t-02
1	• 12060t+01	• 12000t+01	• 12000t+01	• 12496t+01	• 19939t+00	• 58841t-01
1	• 36811t+00	• 34942t+00	• 34942t+00	• 36169t+00	• 97467t+00	• 58130t-03
1	• 18005t+00	• 13673t+00	• 12894t+00	• 16920t+00	• 19496t+00	• 51166t-02
1	• 13000t+01	• 13000t+01	• 13000t+01	• 14000t+01	• 22659t+00	• 58662t-01
1	• 36915t+00	• 37461t+00	• 35169t+00	• 34615t+00	• 36022t+00	• 96878t+00
1	• 12060t+01	• 12060t+01	• 12060t+01	• 12496t+01	• 14762t+00	• 97772t+00
1	• 36811t+00	• 34942t+00	• 33746t+00	• 33746t+00	• 36594t+00	• 50182t+00
2	• 13573t+00	• 91740t-01	• 87140t-01	• 12117t+00	• 10482t+00	• 69886t-01
2	• 12060t+01	• 12060t+01	• 13000t+01	• 14000t+01	• 13495t+01	• 22476t+00
2	• 36811t+00	• 37446t+00	• 37446t+00	• 35169t+00	• 36022t+00	• 97130t+00
2	• 12060t+01	• 12060t+01	• 13000t+01	• 14000t+01	• 13495t+01	• 41042t-01
2	• 36811t+00	• 37446t+00	• 37446t+00	• 35169t+00	• 36022t+00	• 97130t+00
3	• 91740t-01	• 4190t-01	• 3780t-01	• 87140t-01	• 65199t-01	• 49522t-02
3	• 12060t+01	• 12060t+01	• 13000t+01	• 14000t+01	• 14950t+01	• 57032t-01
3	• 37446t+00	• 37446t+00	• 37446t+00	• 35169t+00	• 36022t+00	• 97130t+00
3	• 87140t-01	• 3780t-01	• 3780t-01	• 87140t-01	• 65199t-01	• 49522t-02
3	• 12060t+01	• 12060t+01	• 13000t+01	• 14000t+01	• 14950t+01	• 57032t-01
3	• 37446t+00	• 37446t+00	• 37446t+00	• 35169t+00	• 36022t+00	• 97130t+00
13	• 34940t-01	• 33052t+00				
13	• 12060t+01	• 12060t+01	• 13000t+01	• 14000t+01	• 15495t+01	• 31497t+00
13	• 37446t+00	• 37446t+00	• 37446t+00	• 35169t+00	• 36022t+00	• 101515t-01

M	N	P	Y:	Y:	X:	X:	A:	X:	X:	C4	C5	C6		
Y:	Z:	T:	Y:	Z:	Y:	Z:	Y:	Y:	Z:	FL	FL	FL		
Z:	T:	L:	Z:	T:	Z:	T:	Z:	Y:	Z:	CZ1	CZ1	CZ1		
6	• 413<5t+u0	• 342<7t+u0	• 150<8t+u1	• 154<9t+u1	• 154<9t+u1	• 7051d <u>t</u> -u2	• 202393t+u0	• 202393t+u0						
19	• 150<8t+u1	• 150<8t+u1	• 2<16t+u0	• 2<16t+u0	• 7033t-u1	• 67425t+u0	• 67425t+u0							
20	• 2<16t+u0	• 2<16t+u0	• 3032t+u0	• 3032t+u0	• 1302t-u2	• 64448t-u1	• 64448t-u1							
7	• 37519t+u0	• 31437t+u0	• 140<8t+u1	• 144<9t+u1	• 144<9t+u1	• 56111t <u>t</u> -u2	• 20403t+u0	• 20403t+u0						
18	• 140<8t+u1	• 140<8t+u1	• 31437t+u0	• 31437t+u0	• 6967t-01	• 4671t <u>E</u> +u0	• 4671t <u>E</u> +u0							
20	• 31437t+u0	• 24580t+u0	• 150<8t+u1	• 150<8t+u1	• 91582t <u>t</u> -u3	• 82230t-u1	• 82230t-u1							
7	• 34720t+u0	• 24580t+u0	• 150<8t+u1	• 154<9t+u1	• 154<9t+u1	• 540<3t <u>t</u> -02	• 48379t+u0	• 48379t+u0						
19	• 150<8t+u1	• 24580t+u0	• 24950t+u0	• 2576t <u>d</u> +u0	• 2790t <u>b</u> +u0	• 64107t <u>t</u> -u1	• 22947t+u0	• 22947t+u0						
210	• 24950t+u0	• 31437t+u0	• 150<8t+u1	• 2754<2t+u0	• 32076t <u>t</u> +u0	• 64496t <u>t</u> -03	• 14957t+u0	• 14957t+u0						
8	• 31437t+u0	• 26062t+u0	• 24824t+u0	• 2630t <u>U</u> +u0	• 21204t <u>U</u> +u0	• 5467t <u>t</u> -02	• 44625t+u0	• 44625t+u0						
18	• 140<8t+u1	• 140<8t+u1	• 34037t+u0	• 1500t <u>U</u> +u1	• 14494t <u>U</u> +u1	• 61645t <u>t</u> -u1	• 21442t+u0	• 21442t+u0						
211	• 31437t+u0	• 33017t+u0	• 2455t <u>U</u> +u0	• 31332t <u>U</u> +u0	• 83859t <u>t</u> -u3	• 15549t+u0	• 15549t+u0							
8	• 24580t+u0	• 24824t+u0	• 150<8t+u1	• 2466t <u>U</u> +u0	• 20922t <u>U</u> +u0	• 48941t <u>t</u> -02	• 19676t+u0	• 19676t+u0						
19	• 150<8t+u1	• 24580t+u0	• 30054t+u0	• 1500t <u>U</u> +u1	• 14494t <u>U</u> +u1	• 61820t <u>t</u> -u1	• 5032t <u>U</u> +u0	• 5032t <u>U</u> +u0						
212	• 24580t+u0	• 33047t+u0	• 20741t <u>U</u> +u0	• 92609t <u>U</u> +u0	• 82595t <u>t</u> -u3	• 5076t <u>t</u> -u1	• 5076t <u>t</u> -u1							
9	• 27802t+u0	• 21477t+u0	• 140<8t+u1	• 2476t <u>U</u> +u0	• 25173t <u>U</u> +u0	• 16906t <u>t</u> -02	• 19874t+u0	• 19874t+u0						
18	• 140<8t+u1	• 140<8t+u1	• 34037t+u0	• 1500t <u>U</u> +u1	• 14494t <u>U</u> +u1	• 61730t <u>t</u> -u1	• 40001t+u0	• 40001t+u0						
213	• 33047t+u0	• 31507t+u0	• 30624t <u>U</u> +u0	• 32366t <u>U</u> +u0	• 94401t <u>t</u> +u0	• 5423t <u>t</u> -u1	• 5423t <u>t</u> -u1							
9	• 24824t+u0	• 19725t+u0	• 140<8t+u1	• 2476t <u>U</u> +u0	• 25173t <u>U</u> +u0	• 16906t <u>t</u> -02	• 54874t+u0	• 54874t+u0						
19	• 150<8t+u1	• 150<8t+u1	• 31507t+u0	• 1500t <u>U</u> +u1	• 14494t <u>U</u> +u1	• 61667t <u>t</u> -u1	• 20970t+u0	• 20970t+u0						
214	• 31507t+u0	• 31507t+u0	• 31507t+u0	• 31507t+u0	• 31507t+u0	• 31507t+u0	• 31507t+u0	• 27807t <u>U</u> +u0	• 29089t <u>U</u> +u0	• 81208t <u>t</u> -u3	• 1250t+u0	• 1250t+u0		
10	• 21249t+u0	• 16730t+u0	• 150<8t+u1	• 17475t <u>U</u> +u0	• 16700t <u>U</u> +u0	• 16700t <u>t</u> -02	• 42200t+u0	• 42200t+u0						
18	• 140<8t+u1	• 140<8t+u1	• 34032t+u0	• 1500t <u>U</u> +u1	• 14494t <u>U</u> +u1	• 30567t <u>t</u> -u1	• 20970t+u0	• 20970t+u0						
215	• 34032t+u0	• 30654t+u0	• 31567t <u>U</u> +u0	• 31533t <u>U</u> +u0	• 93723t <u>t</u> +u0	• 78740t <u>t</u> -u1	• 78740t <u>t</u> -u1							
10	• 19725t+u0	• 16730t+u0	• 150<8t+u1	• 16412t <u>U</u> +u0	• 15274t <u>U</u> +u0	• 13274t <u>t</u> -02	• 39066t+u0	• 39066t+u0						
19	• 150<8t+u1	• 150<8t+u1	• 32130t+u0	• 1500t <u>U</u> +u1	• 14494t <u>U</u> +u1	• 20132t <u>U</u> +u0	• 50194t <u>t</u> -01	• 2023t+u0	• 2023t+u0					
216	• 32130t+u0	• 27814t+u0	• 32130t+u0	• 27814t <u>U</u> +u0	• 30343t <u>U</u> +u0	• 94607t <u>t</u> +u0	• 70964t <u>t</u> -03	• 78742t <u>t</u> -03	• 78742t <u>t</u> -03					
10	• 19725t+u0	• 16730t+u0	• 150<8t+u1	• 1689t <u>U</u> +u0	• 15121t <u>U</u> +u0	• 40492t <u>t</u> -02	• 19447t+u0	• 19447t+u0						
19	• 150<8t+u1	• 150<8t+u1	• 32130t+u0	• 1500t <u>U</u> +u1	• 14494t <u>U</u> +u1	• 20132t <u>U</u> +u0	• 50194t <u>t</u> -01	• 2023t+u0	• 2023t+u0					
216	• 32130t+u0	• 27814t+u0	• 32130t+u0	• 27814t <u>U</u> +u0	• 30343t <u>U</u> +u0	• 94607t <u>t</u> +u0	• 70964t <u>t</u> -03	• 78742t <u>t</u> -03	• 78742t <u>t</u> -03					
11	• 169<11t+u0	• 12144t+u0	• 140<8t+u1	• 1296t <u>U</u> +u0	• 99802t <u>U</u> -01	• 48195t <u>t</u> -02	• 19566t+u0	• 19566t+u0						
18	• 140<8t+u1	• 140<8t+u1	• 32130t+u0	• 1500t <u>U</u> +u1	• 14494t <u>U</u> +u1	• 25848t <u>U</u> +u0	• 50495t <u>t</u> -01	• 36499t+u0	• 36499t+u0					
217	• 32130t+u0	• 32130t+u0	• 32130t+u0	• 32130t+u0	• 32130t+u0	• 32130t+u0	• 32130t+u0	• 32130t+u0	• 3567t <u>U</u> +u0	• 72194t <u>t</u> -03	• 35449t <u>t</u> -01	• 71696t <u>t</u> -03	• 71696t <u>t</u> -03	
11	• 15671t+CO	• 11224t+u0	• 16010t+u0	• 14245t <u>U</u> +u0	• 12652t <u>U</u> +u0	• 98604t <u>U</u> -01	• 44674t <u>t</u> -02	• 3402t+u0	• 3402t+u0					
19	• 15CC1t+u1	• 12144t+u0	• 16010t+u0	• 14494t <u>U</u> +u1	• 12442t <u>U</u> +u1	• 2470t <u>U</u> +u0	• 50466t <u>t</u> -01	• 19566t+u0	• 19566t+u0					
218	• 32130t+u0	• 32130t+u0	• 32130t+u0	• 32130t+u0	• 32130t+u0	• 32130t+u0	• 32130t+u0	• 29114t <u>U</u> +u0	• 30804t <u>U</u> +u0	• 94952t <u>t</u> +u0	• 71696t <u>t</u> -03	• 71696t <u>t</u> -03	• 71696t <u>t</u> -03	
12	• 14222t+u0	• 8167t+u1	• 14010t+u1	• 14222t <u>U</u> +u0	• 11811t <u>U</u> +u1	• 69442t <u>U</u> -01	• 36401t <u>t</u> -02	• 14451t+u0	• 14451t+u0					
18	• 14010t+u1	• 14010t+u1	• 32130t+u0	• 14494t <u>U</u> +u1	• 12442t <u>U</u> +u1	• 2561t <u>U</u> +u0	• 5213t <u>t</u> -01	• 19643t+u0	• 19643t+u0					
219	• 32130t+u0	• 32130t+u0	• 32130t+u0	• 32130t+u0	• 32130t+u0	• 32130t+u0	• 32130t+u0	• 32130t+u0	• 34043t <u>U</u> +u0	• 90404t <u>U</u> +u0	• 62942t <u>t</u> -03	• 39005t <u>t</u> -01	• 12570t+u0	• 12570t+u0
12	• 14222t+u0	• 8167t+u1	• 14010t+u1	• 14222t <u>U</u> +u0	• 11811t <u>U</u> +u1	• 69442t <u>U</u> -01	• 36401t <u>t</u> -02	• 14451t+u0	• 14451t+u0					
18	• 14010t+u1	• 14010t+u1	• 32130t+u0	• 14494t <u>U</u> +u1	• 12442t <u>U</u> +u1	• 2561t <u>U</u> +u0	• 5213t <u>t</u> -01	• 19643t+u0	• 19643t+u0					
220	• 32130t+u0	• 32130t+u0	• 32130t+u0	• 32130t+u0	• 32130t+u0	• 32130t+u0	• 32130t+u0	• 32130t+u0	• 34043t <u>U</u> +u0	• 90404t <u>U</u> +u0	• 62942t <u>t</u> -03	• 39005t <u>t</u> -01	• 12570t+u0	• 12570t+u0

	R	Y ₁	Y ₂	Y ₃	Y ₄	X ₁	X ₂	X ₃	X ₄	Y ₁	Y ₂	Y ₃	Y ₄	Z ₁	Z ₂	Z ₃	Z ₄	A	C ₂₄	
	P	Y ₁	Y ₂	Y ₃	Y ₄	X ₁	X ₂	X ₃	X ₄	Y ₁	Y ₂	Y ₃	Y ₄	Z ₁	Z ₂	Z ₃	Z ₄	FL	C ₂₅	
13	*8.8570t-01	*3.7940t-01	*1.9050t+01	*3.5472t+01	*8.8570t-01	*3.7940t-01	*1.9050t+01	*3.5472t+01	*4.0420t-01	*4.4200t-01	*3.4920t-01	*4.4200t-01	*1.9449t+00	-0.19449t-02	-0.19449t+00	-0.19449t-02	-0.19449t+00	-0.19449t-02	-0.19449t+00	
18	*1.4050t+01	*1.4050t+01	*1.4050t+01	*1.4050t+01	*1.4050t+01	*1.4050t+01	*1.4050t+01	*1.4050t+01	*1.4050t+01	*1.4050t+01	*1.4050t+01	*1.4050t+01	*1.4050t+01	*1.4050t+01	*1.4050t+01	*1.4050t+01	*1.4050t+01	*1.4050t+01	*1.4050t+01	
221	*3.5472t+01	*3.5472t+01	*3.5472t+01	*3.5472t+01	*3.5472t+01	*3.5472t+01	*3.5472t+01	*3.5472t+01	*3.5472t+01	*3.5472t+01	*3.5472t+01	*3.5472t+01	*3.5472t+01	*3.5472t+01	*3.5472t+01	*3.5472t+01	*3.5472t+01	*3.5472t+01	*3.5472t+01	
13	*7.5850t-01	*3.4760t-01	*1.5000t+01	*3.4760t-01	*7.5850t-01	*3.4760t-01	*1.5000t+01	*3.4760t-01	*6.7250t-01	*6.7250t-01	*6.7250t-01	*6.7250t-01	*6.7250t-01	*4.1078t-02	*3.7933t+00	*3.7933t+00	*3.7933t+00	*3.7933t+00	*3.7933t+00	
19	*1.5000t+01	*1.5000t+01	*1.5000t+01	*1.5000t+01	*1.5000t+01	*1.5000t+01	*1.5000t+01	*1.5000t+01	*1.5000t+01	*1.5000t+01	*1.5000t+01	*1.5000t+01	*1.5000t+01	*5.0582t-01	*1.4507t+00	*1.4507t+00	*1.4507t+00	*1.4507t+00	*1.4507t+00	
222	*3.4894t+01	*3.4894t+01	*3.4894t+01	*3.4894t+01	*3.4894t+01	*3.4894t+01	*3.4894t+01	*3.4894t+01	*3.4894t+01	*3.4894t+01	*3.4894t+01	*3.4894t+01	*3.4894t+01	*6.7806t-03	*3.0408t-01	*3.0408t-01	*3.0408t-01	*3.0408t-01	*3.0408t-01	
14	*3.7450t-01	*0.5770t+00	*1.4010t+01	*3.7450t-01	*3.7450t-01	*0.5770t+00	*1.4010t+01	*3.7450t-01	*1.6000t-01	*1.6000t-01	*1.6000t-01	*1.6000t-01	*1.6000t-01	*3.7330t-02	*3.0942t+00	*3.0942t+00	*3.0942t+00	*3.0942t+00	*3.0942t+00	
18	*1.4010t+01	*1.4010t+01	*1.4010t+01	*1.4010t+01	*1.4010t+01	*1.4010t+01	*1.4010t+01	*1.4010t+01	*1.4010t+01	*1.4010t+01	*1.4010t+01	*1.4010t+01	*1.4010t+01	*5.4940t-01	*1.4194t+00	*1.4194t+00	*1.4194t+00	*1.4194t+00	*1.4194t+00	
223	*3.5653t+01	*3.5653t+01	*3.5653t+01	*3.5653t+01	*3.5653t+01	*3.5653t+01	*3.5653t+01	*3.5653t+01	*3.5653t+01	*3.5653t+01	*3.5653t+01	*3.5653t+01	*3.5653t+01	*6.3330t-03	*8.7129t-02	*8.7129t-02	*8.7129t-02	*8.7129t-02	*8.7129t-02	
14	*3.4710t-01	*0.6690t+00	*1.5000t+01	*3.4710t-01	*3.4710t-01	*0.6690t+00	*1.5000t+01	*3.4710t-01	*1.6500t-01	*1.6500t-01	*1.6500t-01	*1.6500t-01	*1.6500t-01	*3.4669t-02	*1.4412t+00	*1.4412t+00	*1.4412t+00	*1.4412t+00	*1.4412t+00	
19	*1.5000t+01	*1.5000t+01	*1.5000t+01	*1.5000t+01	*1.5000t+01	*1.5000t+01	*1.5000t+01	*1.5000t+01	*1.5000t+01	*1.5000t+01	*1.5000t+01	*1.5000t+01	*1.5000t+01	*5.0466t-01	*1.7944t+00	*1.7944t+00	*1.7944t+00	*1.7944t+00	*1.7944t+00	
224	*3.5026t+01	*3.5026t+01	*3.5026t+01	*3.5026t+01	*3.5026t+01	*3.5026t+01	*3.5026t+01	*3.5026t+01	*3.5026t+01	*3.5026t+01	*3.5026t+01	*3.5026t+01	*3.5026t+01	*6.3472t-03	*3.3508t-02	*3.3508t-02	*3.3508t-02	*3.3508t-02	*3.3508t-02	

QUESTIONABLE POINT - PUNK FIT	• 1.31E-02	• 1.16E-02	• 1.16E-02	• 1.16E-02
WARNING LUNG THIN QUAD.	• 1.16E-02	• 1.16E-02	• 1.16E-02	• 1.16E-02
5 • 2.35E+00	• 1.16E+00	• 1.16E+00	• 1.16E+00	• 1.16E+00
24 • 1.96E+01	• 1.96E+01	• 1.96E+01	• 1.96E+01	• 1.96E+01
262 • 1.99E+01	• 1.99E+01	• 1.99E+01	• 1.99E+01	• 1.99E+01
6 • 2.7233E+00	• 2.7233E+00	• 2.7233E+00	• 2.7233E+00	• 2.7233E+00
23 • 1.6000E+01	• 1.6000E+01	• 1.6000E+01	• 1.6000E+01	• 1.6000E+01
263 • 1.70E+00	• 1.35544E+00	• 1.35544E+00	• 1.35544E+00	• 1.35544E+00
QUESTIONABLE POINT - PUNK FIT	• 1.31E-02	• 1.16E-02	• 1.16E-02	• 1.16E-02
WARNING LUNG THIN QUAD.	• 1.16E-02	• 1.16E-02	• 1.16E-02	• 1.16E-02
7 • 1.9544E+00	• 1.0456E+00	• 1.0456E+00	• 1.0456E+00	• 1.0456E+00
24 • 1.6000E+01	• 1.6000E+01	• 1.6000E+01	• 1.6000E+01	• 1.6000E+01
264 • 1.6190E+00	• 1.3205E+00	• 1.3205E+00	• 1.3205E+00	• 1.3205E+00
QUESTIONABLE POINT - PUNK FIT	• 1.31E-02	• 1.16E-02	• 1.16E-02	• 1.16E-02
WARNING LUNG THIN QUAD.	• 1.16E-02	• 1.16E-02	• 1.16E-02	• 1.16E-02
7 • 2.29E+00	• 1.16E+00	• 1.16E+00	• 1.16E+00	• 1.16E+00
23 • 1.6000E+01	• 1.6000E+01	• 1.6000E+01	• 1.6000E+01	• 1.6000E+01
265 • 1.6544E+00	• 1.19444E+00	• 1.19444E+00	• 1.19444E+00	• 1.19444E+00
QUESTIONABLE POINT - PUNK FIT	• 1.31E-02	• 1.16E-02	• 1.16E-02	• 1.16E-02
WARNING LUNG THIN QUAD.	• 1.16E-02	• 1.16E-02	• 1.16E-02	• 1.16E-02
7 • 1.6405E+00	• 1.1349E+00	• 1.1349E+00	• 1.1349E+00	• 1.1349E+00
24 • 1.6000E+01	• 1.6000E+01	• 1.6000E+01	• 1.6000E+01	• 1.6000E+01
266 • 1.3232E+00	• 1.13494E+00	• 1.13494E+00	• 1.13494E+00	• 1.13494E+00
QUESTIONABLE POINT - PUNK FIT	• 1.31E-02	• 1.16E-02	• 1.16E-02	• 1.16E-02
WARNING LUNG THIN QUAD.	• 1.16E-02	• 1.16E-02	• 1.16E-02	• 1.16E-02
8 • 1.9494E+00	• 1.0359E+00	• 1.0359E+00	• 1.0359E+00	• 1.0359E+00
23 • 1.8000E+01	• 1.0600E+01	• 1.0600E+01	• 1.0600E+01	• 1.0600E+01
267 • 1.9494E+00	• 1.2020E+00	• 1.2020E+00	• 1.2020E+00	• 1.2020E+00
QUESTIONABLE POINT - PUNK FIT	• 1.31E-02	• 1.16E-02	• 1.16E-02	• 1.16E-02
WARNING LUNG THIN QUAD.	• 1.16E-02	• 1.16E-02	• 1.16E-02	• 1.16E-02
8 • 1.3964E+00	• 1.1714E+00	• 1.1714E+00	• 1.1714E+00	• 1.1714E+00
24 • 1.9000E+01	• 1.1200E+01	• 1.1200E+01	• 1.1200E+01	• 1.1200E+01
268 • 1.3964E+00	• 1.4471E+00	• 1.4471E+00	• 1.4471E+00	• 1.4471E+00
QUESTIONABLE POINT - PUNK FIT	• 1.31E-02	• 1.16E-02	• 1.16E-02	• 1.16E-02
WARNING LUNG THIN QUAD.	• 1.16E-02	• 1.16E-02	• 1.16E-02	• 1.16E-02
9 • 1.6359E+00	• 1.1219E+00	• 1.1219E+00	• 1.1219E+00	• 1.1219E+00
23 • 1.6000E+01	• 1.6000E+01	• 1.6000E+01	• 1.6000E+01	• 1.6000E+01
269 • 2.0201E+00	• 1.4471E+00	• 1.4471E+00	• 1.4471E+00	• 1.4471E+00
QUESTIONABLE POINT - PUNK FIT	• 1.31E-02	• 1.16E-02	• 1.16E-02	• 1.16E-02
WARNING LUNG THIN QUAD.	• 1.16E-02	• 1.16E-02	• 1.16E-02	• 1.16E-02
10 • 1.999E+00	• 1.0327E+00	• 1.0327E+00	• 1.0327E+00	• 1.0327E+00
23 • 1.6000E+01	• 1.6000E+01	• 1.6000E+01	• 1.6000E+01	• 1.6000E+01
271 • 2.0803E+00	• 2.2117E+00	• 2.2117E+00	• 2.2117E+00	• 2.2117E+00

SOLID STATE = LIQUID
XY2 POTENTIAL FLUID REGION AND LIQUID REGION
SAMPLE PRELIMINARY EVALUATION

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300	301	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320	321	322	323	324	325	326	327	328	329	330	331	332	333	334	335	336	337	338	339	340	341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360	361	362	363	364	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379	380	381	382	383	384	385	386	387	388	389	390	391	392	393	394	395	396	397	398	399	400	401	402	403	404	405	406	407	408	409	410	411	412	413	414	415	416	417	418	419	420	421	422	423	424	425	426	427	428	429	430	431	432	433	434	435	436	437	438	439	440	441	442	443	444	445	446	447	448	449	450	451	452	453	454	455	456	457	458	459	460	461	462	463	464	465	466	467	468	469	470	471	472	473	474	475	476	477	478	479	480	481	482	483	484	485	486	487	488	489	490	491	492	493	494	495	496	497	498	499	500	501	502	503	504	505	506	507	508	509	510	511	512	513	514	515	516	517	518	519	520	521	522	523	524	525	526	527	528	529	530	531	532	533	534	535	536	537	538	539	540	541	542	543	544	545	546	547	548	549	550	551	552	553	554	555	556	557	558	559	560	561	562	563	564	565	566	567	568	569	570	571	572	573	574	575	576	577	578	579	580	581	582	583	584	585	586	587	588	589	590	591	592	593	594	595	596	597	598	599	600	601	602	603	604	605	606	607	608	609	610	611	612	613	614	615	616	617	618	619	620	621	622	623	624	625	626	627	628	629	630	631	632	633	634	635	636	637	638	639	640	641	642	643	644	645	646	647	648	649	650	651	652	653	654	655	656	657	658	659	660	661	662	663	664	665	666	667	668	669	670	671	672	673	674	675	676	677	678	679	680	681	682	683	684	685	686	687	688	689	690	691	692	693	694	695	696	697	698	699	700	701	702	703	704	705	706	707	708	709	710	711	712	713	714	715	716	717	718	719	720	721	722	723	724	725	726	727	728	729	730	731	732	733	734	735	736	737	738	739	740	741	742	743	744	745	746	747	748	749	750	751	752	753	754	755	756	757	758	759	760	761	762	763	764	765	766	767	768	769	770	771	772	773	774	775	776	777	778	779	780	781	782	783	784	785	786	787	788	789	790	791	792	793	794	795	796	797	798	799	800	801	802	803	804	805	806	807	808	809	810	811	812	813	814	815	816	817	818	819	820	821	822	823	824	825	826	827	828	829	830	831	832	833	834	835	836	837	838	839	840	841	842	843	844	845	846	847	848	849	850	851	852	853	854	855	856	857	858	859	860	861	862	863	864	865	866	867	868	869	870	871	872	873	874	875	876	877	878	879	880	881	882	883	884	885	886	887	888	889	890	891	892	893	894	895	896	897	898	899	900	901	902	903	904	905	906	907	908	909	910	911	912	913	914	915	916	917	918	919	920	921	922	923	924	925	926	927	928	929	930	931	932	933	934	935	936	937	938	939	940	941	942	943	944	945	946	947	948	949	950	951	952	953	954	955	956	957	958	959	960	961	962	963	964	965	966	967	968	969	970	971	972	973	974	975	976	977	978	979	980	981	982	983	984	985	986	987	988	989	990	991	992	993	994	995	996	997	998	999	1000
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XYZ POTENTIAL FIELD PROGRAM SECTION 4

SAMPLE PROBLEM TRIAXIAL ELLIPSOID

 $X \text{ VELOCITY} = -1.0$

ITERATION	SUM OF CHANNELS	A	B	C
1	-26052E+00			
2	-13219E+00			
3	-67071E-01			
4	-51901E-01			
5	-44774E-02			
6	-35685E-02			
7	-13436E-02			
8	-20600E-02			

 $X \text{ VELOCITY} = +0$

ITERATION	SUM OF CHANNELS	A	B	C
1	-11424E+01			
2	-2740E+01			
3	-16524E+01			
4	-12150E+01			
5	-69020E+00			
6	-64920E+00			
7	-47463E+00			
8	-34694E+00			
9	-25362E+00			
10	-18540E+00			
11	-13005E-01			

 $X \text{ VELOCITY} = +0$

ITERATION	SUM OF CHANNELS	A	B	C
1	-15305E+02			
2	-34163E+01			
3	-75052E+00			
4	-16720E+00			
5	-36673E-01			
6	-81712E-02			
7	-13054E-02			

 $X \text{ VELOCITY} = +0$

ITERATION	SUM OF CHANNELS	A	B	C
1	-15305E+02			
2	-34163E+01			
3	-75052E+00			
4	-16720E+00			
5	-36673E-01			
6	-81712E-02			
7	-13054E-02			

XYZ POTENTIAL FIELD PROGRAM SECTION 4

SAMPLE PROBLEM TRIAXIAL ELLIPSOID

SAMPLE PREPARATION INFLUENCE

PT.	A. Flux	V	V/L	AUD/V	CP	SOURCE	V NORMAL
1	9.63639	0.614777	0.1665	0.28104	0.92068	• 0.11624	• 1.2E-04
2	9.66611	0.614777	0.1665	0.28104	0.91830	• 0.11616	• 1.2E-04
3	9.62666	0.614777	0.1665	0.28104	0.91297	• 0.98884	• 1.1E-04
4	9.63500	0.614777	0.1665	0.28104	0.91217	• 0.98733	• 1.1E-04
5	8.65211	0.64949	0.24664	0.05004	0.12012	• 0.74880	• 9.9E-05
6	8.6304	0.64949	0.24664	0.05004	0.12134	• 0.74850	• 9.9E-05
7	7.679.4	0.64949	0.24664	0.05004	0.42606	• 0.5888	• 7.4E-05
8	7.6751	0.64949	0.24577	0.05000	0.42917	• 0.5887	• 7.4E-05
9	6.6989	0.49949	0.36954	0.04442	0.4386	• 0.4587	• 6.2E-05
10	6.6881	0.49949	0.36954	0.04442	0.43807	• 0.4586	• 6.2E-05
11	5.6471	0.49949	0.40706	0.03280	0.42664	• 0.3045	• 5.0E-05
12	5.5737	0.49949	0.40664	0.03280	0.42664	• 0.3045	• 5.0E-05
13	4.6538	0.49949	0.43577	0.02947	0.3548	• 0.28450	• 4.1E-05
14	4.6477	0.49949	0.43577	0.02947	0.3537	• 0.28430	• 4.1E-05
15	4.1110	0.41110	0.45545	0.02737	0.30145	• 0.28133	• 3.4E-05
16	4.0997	0.40997	0.45545	0.02737	0.30145	• 0.28133	• 3.4E-05
17	3.3665	0.49949	0.47006	0.02610	0.32438	• 0.28110	• 2.7E-05
18	3.3571	0.49949	0.46566	0.02610	0.32428	• 0.28110	• 2.7E-05
19	2.6741	0.49949	0.41121	0.02316	0.31948	• 0.28110	• 2.2E-05
20	2.6674	0.49949	0.40006	0.02316	0.31917	• 0.28110	• 2.2E-05
21	2.6317	0.49949	0.40517	0.02316	0.30166	• 0.28110	• 2.2E-05
22	2.6266	0.49949	0.40715	0.02316	0.30166	• 0.28110	• 2.2E-05
23	1.4208	0.49949	0.43166	0.02066	0.30536	• 0.40021	• 9.0E-06
24	1.4173	0.49949	0.43066	0.02066	0.30523	• 0.40021	• 9.0E-06
25	0.8322	0.49949	0.47593	0.01958	0.30877	• 0.40021	• 9.0E-06
26	0.8332	0.49949	0.47645	0.01958	0.30877	• 0.40021	• 9.0E-06
27	0.6622	0.49949	0.47954	0.01958	0.30877	• 0.40021	• 9.0E-06
28	0.6616	0.49949	0.48034	0.01958	0.30877	• 0.40021	• 9.0E-06
29	9.6144	0.24995	0.12111	0.00240	0.52525	• 0.27423	• 6.7E-06
30	9.7372	0.34993	0.12063	0.00240	0.273348	• 0.20114	• 1.2E-04
31	9.3556	0.24997	0.12045	0.00240	0.28779	• 0.19085	• 6.7E-06
32	9.2659	0.34992	0.12032	0.00240	0.3112	• 0.18932	• 1.2E-04
33	8.65809	0.24945	0.12026	0.00240	0.34441	• 0.18932	• 1.2E-04
34	8.6211	0.34992	0.12021	0.00240	0.34441	• 0.18932	• 1.2E-04
35	7.6344	0.24945	0.12016	0.00240	0.3609	• 0.18932	• 1.2E-04
36	7.5759	0.34992	0.11133	0.00240	0.4086	• 0.19042	• 1.2E-04
37	6.66484	0.24945	0.36676	0.00240	0.4244	• 0.19042	• 1.2E-04
38	6.6475	0.34992	0.30349	0.00240	0.42701	• 0.19042	• 1.2E-04
39	5.7035	0.24945	0.40429	0.00240	0.43699	• 0.19042	• 1.2E-04
40	5.66602	0.34992	0.40149	0.00240	0.43699	• 0.19042	• 1.2E-04
41	4.6232	0.24945	0.43216	0.00240	0.43699	• 0.19042	• 1.2E-04
42	4.7863	0.34992	0.42445	0.00240	0.43699	• 0.19042	• 1.2E-04
43	4.6740	0.24945	0.42132	0.00240	0.43699	• 0.19042	• 1.2E-04
44	4.0479	0.34992	0.44626	0.00240	0.43699	• 0.19042	• 1.2E-04
45	3.3401	0.24945	0.40631	0.00240	0.43699	• 0.19042	• 1.2E-04
46	3.3145	0.34992	0.40631	0.00240	0.42465	• 0.19120	• 2.7E-05
47	2.6534	0.24945	0.41705	0.00240	0.42465	• 0.19120	• 2.7E-05
48	2.6335	0.34992	0.41575	0.00240	0.42465	• 0.19120	• 2.7E-05
49	2.0164	0.24945	0.40527	0.00240	0.42465	• 0.19120	• 2.7E-05
50	2.00619	0.34992	0.40527	0.00240	0.42465	• 0.19120	• 2.7E-05

PLATE	X	Y	Z	A ADD.	V SOURCE	V NUMERAL
21	-1.4161	-0.24149	-0.49064	-1.00141	-0.90777	-0.12E-02
22	-0.15943	-0.34473	-0.49064	-0.40032	-0.90104	-0.12E-02
23	-0.02490	-0.24149	-0.49064	-0.40032	-0.90244	-0.61E-02
24	-0.02246	-0.34473	-0.49064	-0.40032	-0.90243	-0.61E-02
25	-0.02003	-0.44905	-0.49064	-0.40032	-0.90234	-0.21E-02
56	-0.25053	-0.34473	-0.49116	-0.40032	-0.90935	-0.0143
57	-0.91362	-0.49150	-0.49064	-0.0002	-0.90938	-0.21E-02
28	-0.50044	-0.54738	-0.49493	-0.0002	-0.94627	-0.12E-04
29	-0.91476	-0.44190	-0.52117	-0.0002	-0.80318	-0.11534
60	-0.90657	-0.54948	-0.50044	-0.0002	-0.09824	-0.10E-04
61	-0.84326	-0.49493	-0.50044	-0.0002	-0.09790	-0.10E-04
62	-0.83203	-0.49150	-0.50044	-0.0002	-0.07459	-0.94E-02
63	-0.74973	-0.44910	-0.50253	-0.0002	-0.11364	-0.94E-02
64	-0.73973	-0.54948	-0.50253	-0.0002	-0.11364	-0.74E-02
65	-0.65290	-0.44949	-0.50117	-0.0002	-0.11364	-0.41E-02
66	-0.64424	-0.54948	-0.50117	-0.0002	-0.11364	-0.41E-02
67	-0.5014	-0.44990	-0.52117	-0.0002	-0.11364	-0.41E-02
68	-0.55271	-0.54948	-0.52117	-0.0002	-0.11364	-0.41E-02
69	-0.41367	-0.44990	-0.42500	-0.0002	-0.11364	-0.41E-02
70	-0.40738	-0.54948	-0.42500	-0.0002	-0.11364	-0.41E-02
71	-0.40057	-0.44949	-0.44351	-0.0002	-0.11364	-0.41E-02
72	-0.34529	-0.54948	-0.44351	-0.0002	-0.11364	-0.41E-02
73	-0.32801	-0.44990	-0.45314	-0.0002	-0.11364	-0.41E-02
74	-0.32306	-0.54948	-0.45220	-0.0002	-0.11364	-0.41E-02
75	-0.26062	-0.44990	-0.46900	-0.0002	-0.11364	-0.41E-02
76	-0.27716	-0.54948	-0.46245	-0.0002	-0.11364	-0.41E-02
77	-0.14802	-0.44949	-0.47073	-0.0002	-0.11364	-0.41E-02
78	-0.14539	-0.54948	-0.47073	-0.0002	-0.11364	-0.41E-02
79	-0.13948	-0.44949	-0.48156	-0.0002	-0.11364	-0.41E-02
80	-0.13064	-0.54948	-0.48156	-0.0002	-0.11364	-0.41E-02
81	-0.08141	-0.44949	-0.49537	-0.0002	-0.11364	-0.41E-02
82	-0.02032	-0.54948	-0.49537	-0.0002	-0.11364	-0.41E-02
83	-0.02556	-0.44949	-0.48660	-0.0002	-0.11364	-0.41E-02
84	-0.02622	-0.54948	-0.48660	-0.0002	-0.11364	-0.41E-02
85	-0.93526	-0.44949	-0.49602	-0.0002	-0.11364	-0.41E-02
86	-0.91676	-0.74362	-0.49766	-0.0002	-0.11364	-0.41E-02
87	-0.84172	-0.54705	-0.49770	-0.0002	-0.11364	-0.41E-02
88	-0.87408	-0.74908	-0.49477	-0.0002	-0.11364	-0.41E-02
89	-0.81845	-0.64905	-0.52312	-0.0002	-0.11364	-0.41E-02
90	-0.80226	-0.74902	-0.22851	-0.0002	-0.11364	-0.41E-02
91	-0.72707	-0.64905	-0.24908	-0.0002	-0.11364	-0.41E-02
92	-0.71327	-0.74902	-0.24916	-0.0002	-0.11364	-0.41E-02
93	-0.63369	-0.64905	-0.34917	-0.0002	-0.11364	-0.41E-02
94	-0.62115	-0.74902	-0.34924	-0.0002	-0.11364	-0.41E-02
95	-0.54366	-0.64905	-0.36054	-0.0002	-0.11364	-0.41E-02
96	-0.53240	-0.74902	-0.36054	-0.0002	-0.11364	-0.41E-02
97	-0.45973	-0.64905	-0.42449	-0.0002	-0.11364	-0.41E-02
98	-0.42063	-0.74902	-0.42449	-0.0002	-0.11364	-0.41E-02
99	-0.30874	-0.64905	-0.45046	-0.0002	-0.11364	-0.41E-02
100	-0.31110	-0.74902	-0.46134	-0.0002	-0.11364	-0.41E-02

SAMPLE PREDICTION FOR THE FIRST PREDATOR

A. PLUS	V	V _{MAX}	V	V _{MAX}
P+				
1.01	-0.64460, +0.71402	-0.64460, +0.71402	-0.64460, +0.71402	-0.64460, +0.71402
10.2	-0.34111, +0.37602	-0.34111, +0.37602	-0.34111, +0.37602	-0.34111, +0.37602
103	-0.20745, +0.24795	-0.20745, +0.24795	-0.20745, +0.24795	-0.20745, +0.24795
104	-0.19219, +0.20455	-0.19219, +0.20455	-0.19219, +0.20455	-0.19219, +0.20455
105	-0.16834, +0.17402	-0.16834, +0.17402	-0.16834, +0.17402	-0.16834, +0.17402
106	-0.13440, +0.14175	-0.13440, +0.14175	-0.13440, +0.14175	-0.13440, +0.14175
107	-0.11175, +0.11961	-0.11175, +0.11961	-0.11175, +0.11961	-0.11175, +0.11961
108	-0.09615, +0.10505	-0.09615, +0.10505	-0.09615, +0.10505	-0.09615, +0.10505
109	-0.08109, +0.08745	-0.08109, +0.08745	-0.08109, +0.08745	-0.08109, +0.08745
110	-0.06745, +0.07135	-0.06745, +0.07135	-0.06745, +0.07135	-0.06745, +0.07135
111	-0.05401, +0.05905	-0.05401, +0.05905	-0.05401, +0.05905	-0.05401, +0.05905
112	-0.04142, +0.04702	-0.04142, +0.04702	-0.04142, +0.04702	-0.04142, +0.04702
113	-0.02951, +0.03478	-0.02951, +0.03478	-0.02951, +0.03478	-0.02951, +0.03478
114	-0.01919, +0.02414	-0.01919, +0.02414	-0.01919, +0.02414	-0.01919, +0.02414
115	-0.01037, +0.01498	-0.01037, +0.01498	-0.01037, +0.01498	-0.01037, +0.01498
116	-0.002968, +0.004614	-0.002968, +0.004614	-0.002968, +0.004614	-0.002968, +0.004614
117	-0.78355, +0.84912	-0.78355, +0.84912	-0.78355, +0.84912	-0.78355, +0.84912
118	-0.76151, +0.84914	-0.76151, +0.84914	-0.76151, +0.84914	-0.76151, +0.84914
119	-0.69646, +0.74978	-0.69646, +0.74978	-0.69646, +0.74978	-0.69646, +0.74978
120	-0.67704, +0.64974	-0.67704, +0.64974	-0.67704, +0.64974	-0.67704, +0.64974
121	-0.63651, +0.54716	-0.63651, +0.54716	-0.63651, +0.54716	-0.63651, +0.54716
122	-0.58900, +0.50344	-0.58900, +0.50344	-0.58900, +0.50344	-0.58900, +0.50344
123	-0.52034, +0.44978	-0.52034, +0.44978	-0.52034, +0.44978	-0.52034, +0.44978
124	-0.50563, +0.44974	-0.50563, +0.44974	-0.50563, +0.44974	-0.50563, +0.44974
125	-0.44001, +0.39070	-0.44001, +0.39070	-0.44001, +0.39070	-0.44001, +0.39070
126	-0.42774, +0.39474	-0.42774, +0.39474	-0.42774, +0.39474	-0.42774, +0.39474
127	-0.37211, +0.34978	-0.37211, +0.34978	-0.37211, +0.34978	-0.37211, +0.34978
128	-0.36174, +0.34974	-0.36174, +0.34974	-0.36174, +0.34974	-0.36174, +0.34974
129	-0.30471, +0.29497	-0.30471, +0.29497	-0.30471, +0.29497	-0.30471, +0.29497
130	-0.24621, +0.24975	-0.24621, +0.24975	-0.24621, +0.24975	-0.24621, +0.24975
131	-0.21201, +0.21200	-0.21201, +0.21200	-0.21201, +0.21200	-0.21201, +0.21200
132	-0.23535, +0.23533	-0.23535, +0.23533	-0.23535, +0.23533	-0.23535, +0.23533
133	-0.10395, +0.09714	-0.10395, +0.09714	-0.10395, +0.09714	-0.10395, +0.09714
134	-0.11782, +0.09714	-0.11782, +0.09714	-0.11782, +0.09714	-0.11782, +0.09714
135	-0.12864, +0.09714	-0.12864, +0.09714	-0.12864, +0.09714	-0.12864, +0.09714
136	-0.16565, +0.09715	-0.16565, +0.09715	-0.16565, +0.09715	-0.16565, +0.09715
137	-0.07562, +0.04978	-0.07562, +0.04978	-0.07562, +0.04978	-0.07562, +0.04978
138	-0.07352, +0.04974	-0.07352, +0.04974	-0.07352, +0.04974	-0.07352, +0.04974
139	-0.02374, +0.04978	-0.02374, +0.04978	-0.02374, +0.04978	-0.02374, +0.04978
140	-0.02308, +0.04974	-0.02308, +0.04974	-0.02308, +0.04974	-0.02308, +0.04974
141	-0.084159, +0.04375	-0.084159, +0.04375	-0.084159, +0.04375	-0.084159, +0.04375
142	-0.070876, +0.04205	-0.070876, +0.04205	-0.070876, +0.04205	-0.070876, +0.04205
143	-0.07352, +0.04205	-0.07352, +0.04205	-0.07352, +0.04205	-0.07352, +0.04205
144	-0.02374, +0.04210	-0.02374, +0.04210	-0.02374, +0.04210	-0.02374, +0.04210
145	-0.075648, +0.04210	-0.075648, +0.04210	-0.075648, +0.04210	-0.075648, +0.04210
146	-0.070793, +0.041954	-0.070793, +0.041954	-0.070793, +0.041954	-0.070793, +0.041954
147	-0.065479, +0.041950	-0.065479, +0.041950	-0.065479, +0.041950	-0.065479, +0.041950
148	-0.062940, +0.041954	-0.062940, +0.041954	-0.062940, +0.041954	-0.062940, +0.041954
149	-0.057022, +0.041950	-0.057022, +0.041950	-0.057022, +0.041950	-0.057022, +0.041950
150	-0.048111, +0.041954	-0.048111, +0.041954	-0.048111, +0.041954	-0.048111, +0.041954

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Y		X		Z	
P.T.	Y	X	Z	Abs. Y	Abs. Z
• 5.5674	1.044435	• 1.044435	• 0.11475	-• 2.4290	• 0.7001
201	• 1.05011	• 1.05011	• 0.11475	-• 2.4290	• 0.7001
202	• 1.054147	• 1.054147	• 0.11475	-• 2.7846	• 0.705
203	• 5.5236	1.044435	• 2.42776	• 2.22208	• 6.33E-05
204	• 4.5566	1.044435	• 2.42776	• 2.22208	• 6.33E-05
205	• 4.6126	1.044435	• 2.42776	• 2.22208	• 6.33E-05
206	• 4.4231	1.044435	• 2.53340	• 2.42776	• 6.33E-05
207	• 3.9572	1.044435	• 2.66710	• 4.6324	• 5.4E-05
208	• 3.6360	1.044435	• 2.57747	• 4.0240	• 4.4E-05
209	• 3.3463	1.044435	• 3.00247	• 3.90498	• 4.4E-05
210	• 3.0546	1.044435	• 2.75142	• 2.66710	• 4.4E-05
211	• 2.6350	1.044435	• 3.3332	• 1.9253	• 3.1E-05
212	• 2.2959	1.044435	• 2.6741	• 1.3247	• 3.1E-05
213	• 2.9173	1.044435	• 3.2366	• 0.9400	• 2.5E-05
214	• 2.1257	1.044435	• 2.7669	• 1.3423	• 3.0E-05
215	• 1.8412	1.044435	• 3.5133	• 1.0505	• 3.0E-05
216	• 1.6899	1.044435	• 3.5133	• 1.0505	• 3.0E-05
217	• 1.5957	1.044435	• 3.5133	• 1.0505	• 3.0E-05
218	• 1.4822	1.044435	• 3.5133	• 1.0505	• 3.0E-05
219	• 0.9763	1.044435	• 3.4943	• 1.3423	• 3.0E-05
220	• 0.6974	1.044435	• 3.1228	• 0.9400	• 2.5E-05
221	• 0.5751	1.044435	• 3.4269	• 1.0505	• 3.0E-05
222	• 0.5275	1.044435	• 3.4435	• 1.0505	• 3.0E-05
223	• 0.1826	1.044435	• 3.4363	• 1.0505	• 3.0E-05
224	• 0.1626	1.044435	• 3.1539	• 1.0505	• 3.0E-05
225	• 5.5815	1.044435	• 6.2478	• 5.5149	• 9.2E-05
226	• 4.7761	1.044435	• 6.2478	• 5.5149	• 9.2E-05
227	• 5.3217	1.044435	• 6.2478	• 5.5149	• 9.2E-05
228	• 4.5537	1.044435	• 6.2478	• 5.5149	• 9.2E-05
229	• 4.0844	1.044435	• 1.9413	• 3.9157	• 7.3E-05
230	• 4.1745	1.044435	• 11.867	• 4.9519	• 6.4E-05
231	• 4.3446	1.044435	• 1.64949	• 4.9454	• 6.4E-05
232	• 3.7153	1.044435	• 1.5273	• 4.2157	• 6.4E-05
233	• 3.7818	1.044435	• 2.0862	• 2.0541	• 5.1E-05
234	• 3.2360	1.044435	• 1.7251	• 2.4769	• 4.4E-05
235	• 3.2445	1.044435	• 2.5014	• 1.8203	• 4.4E-05
240	• 2.7762	1.044435	• 1.7635	• 3.4204	• 4.0E-05
241	• 2.7436	1.044435	• 2.4617	• 3.1242	• 3.2E-05
242	• 2.3477	1.044435	• 2.6034	• 1.5174	• 2.4E-05
243	• 2.3262	1.044435	• 2.2639	• 1.5207	• 2.4E-05
244	• 1.9854	1.044435	• 2.1962	• 1.2247	• 1.4E-05
245	• 1.1470	1.044435	• 2.7603	• 1.0504	• 1.4E-05
246	• 0.9415	1.044435	• 2.26420	• 1.3044	• 1.4E-05
247	• 0.6021	1.044435	• 2.2777	• 1.3072	• 1.4E-05
248	• 0.6844	1.044435	• 2.1912	• 1.3072	• 1.4E-05
249	• 0.4715	1.044435	• 2.0597	• 1.3072	• 1.4E-05
250	• 0.4035	1.044435	• 2.4344	• 1.40164	• 1.57E-05

Singer - PRACTICAL USES OF THE X-RAY IN PRACTICE

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SAMPLE PAGE

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PT.	X	Y	Z	Abs. Y	VZ	VY	VP	SOURCE	V NORMA
1	0.9619	0.9619	-0.1454	0.0004	-1.01454	-0.0004	-0.21247	0.00118	-0.14E-06
2	0.9621	0.9621	-0.1454	0.0004	-1.01454	-0.0004	-0.21258	0.00154	-0.56E-06
3	0.9426	0.9426	-0.1454	0.0004	-1.01454	-0.0004	-0.21256	0.00103	-0.16E-06
4	0.9405	0.9405	-0.1454	0.0004	-1.01454	-0.0004	-0.21252	0.00149	-0.48E-06
5	0.9657	0.9657	-0.1454	0.0004	-1.01454	-0.0004	-0.21250	0.00087	-0.14E-06
6	0.9610	0.9610	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.00261	-0.43E-06
7	0.7692	0.7692	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.00078	-0.12E-06
8	0.7673	0.7673	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.00231	-0.30E-06
9	0.6696	0.6696	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.0064	-0.11E-06
10	0.6691	0.6691	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.00206	-0.34E-06
11	0.5747	0.5747	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.00066	-0.93E-04
12	0.5732	0.5732	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.0194	-0.20E-06
13	0.4559	0.4559	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.0062	-0.85E-09
14	0.4547	0.4547	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.0184	-0.22E-06
15	0.4116	0.4116	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.0060	-0.80E-04
16	0.4097	0.4097	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.0174	-0.24E-06
17	0.3365	0.3365	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.0054	-0.72E-04
18	0.3357	0.3357	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.0175	-0.22E-06
19	0.2274	0.2274	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.0050	-0.75E-04
20	0.2667	0.2667	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.0170	-0.22E-06
21	0.2031	0.2031	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.0055	-0.71E-04
22	0.2026	0.2026	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.0164	-0.21E-06
23	0.1420	0.1420	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.0056	-0.70E-04
24	0.1417	0.1417	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.0167	-0.21E-06
25	0.0352	0.0352	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.0055	-0.64E-04
26	0.0353	0.0353	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.0167	-0.20E-06
27	0.0652	0.0652	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.0055	-0.64E-04
28	0.0616	0.0616	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.0166	-0.20E-06
29	0.0612	0.0612	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.0167	-0.21E-06
30	0.9737	0.9737	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.0034	-0.13E-07
31	0.9355	0.9355	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.0034	-0.80E-08
32	0.9263	0.9263	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.0075	-0.11E-07
33	0.6597	0.6597	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.00437	-0.71E-08
34	0.6597	0.6597	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.00491	-0.71E-08
35	0.7634	0.7634	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.0036	-0.45E-08
36	0.7575	0.7575	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.0061	-0.50E-08
37	0.6640	0.6640	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.00389	-0.60E-08
38	0.6597	0.6597	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.00648	-0.80E-08
39	0.5703	0.5703	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.00634	-0.50E-08
40	0.5660	0.5660	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.00634	-0.50E-08
41	0.4523	0.4523	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.00648	-0.80E-08
42	0.4786	0.4786	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.00437	-0.50E-08
43	0.4074	0.4074	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.00491	-0.71E-08
44	0.4047	0.4047	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.0036	-0.34E-08
45	0.3340	0.3340	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.00423	-0.53E-08
46	0.3314	0.3314	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.00294	-0.37E-08
47	0.2655	0.2655	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.0014	-0.20E-08
48	0.2633	0.2633	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.00437	-0.50E-08
49	0.2016	0.2016	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.00300	-0.34E-08
50	0.2009	0.2009	-0.1454	0.0004	-1.01454	-0.0004	-0.21249	0.00282	-0.33E-08

PT.	PLN	V	V	SOURCE	CP
151	*4 0 0 2 1	1.0 0 4 6 3	*1.0 0 4 6 3	*0 1 6 4 6	*0 1 6 4 6
152	*6 1 C 2 4	1.1 4 5 6 4	-1.0 0 4 6 3	*0 1 6 4 6	*0 1 6 4 6
153	*4 1 3 6 3	1.0 3 4 7 8	*0 1 6 4 6	*0 1 6 4 6	*0 1 6 4 6
154	*3 9 7 5 5	1.0 1 4 6 4	*0 1 6 4 6	*0 1 6 4 6	*0 1 6 4 6
155	*3 4 6 6 5	1.0 0 4 9 6	*0 1 6 4 6	*0 1 6 4 6	*0 1 6 4 6
156	*3 3 6 6 7	1.1 4 6 4	*0 1 6 4 6	*0 1 6 4 6	*0 1 6 4 6
157	*2 6 6 4 7	1.0 4 9 7 0	*0 1 6 4 6	*0 1 6 4 6	*0 1 6 4 6
158	*2 7 5 3 7	1.1 4 6 4	*0 1 6 4 6	*0 1 6 4 6	*0 1 6 4 6
159	*2 7 6 2 2	1.0 4 9 7 0	*0 1 6 4 6	*0 1 6 4 6	*0 1 6 4 6
160	*2 4 8 7 9	1.1 4 6 4	*0 1 6 4 6	*0 1 6 4 6	*0 1 6 4 6
161	*1 7 2 4 4	1.0 4 9 7 0	*0 1 6 4 6	*0 1 6 4 6	*0 1 6 4 6
162	*1 6 6 4 4	1.1 4 6 4	*0 1 6 4 6	*0 1 6 4 6	*0 1 6 4 6
163	*1 0 5 4	1.0 4 9 7 0	*0 1 6 4 6	*0 1 6 4 6	*0 1 6 4 6
164	*1 6 2 5	1.1 4 6 4	*0 1 6 4 6	*0 1 6 4 6	*0 1 6 4 6
165	*0 7 1 1 0	1.0 4 9 7 0	*0 1 6 4 6	*0 1 6 4 6	*0 1 6 4 6
166	*0 6 3 3 4	1.1 4 6 4	*0 1 6 4 6	*0 1 6 4 6	*0 1 6 4 6
167	*0 2 3 2	1.0 4 9 7 0	*0 1 6 4 6	*0 1 6 4 6	*0 1 6 4 6
168	*0 2 1 4 6	1.1 4 6 4	*0 1 6 4 6	*0 1 6 4 6	*0 1 6 4 6
169	*7 7 1 7 9	1.2 4 9 5 7	*0 1 6 4 6	*0 1 6 4 6	*0 1 6 4 6
170	*7 2 9 3 9	1.3 4 9 4 8	*0 3 7 7 1	*0 3 7 7 0	*0 3 7 7 0
171	*7 5 6 7	1.2 4 9 5 7	*1 2 1 0 5	*1 2 1 0 5	*1 2 1 0 5
172	*6 5 5 4	1.3 4 9 4 8	*1 4 2 1 6	*1 4 2 1 6	*1 4 2 1 6
173	*6 1 5 4 0	1.2 4 9 5 7	*1 7 2 3 6	*1 7 2 3 6	*1 7 2 3 6
174	*6 3 8 2 9	1.3 4 9 4 8	*1 0 1 6 1	*2 2 7 1 7	*2 2 7 1 7
175	*6 0 0 4 9	1.2 4 9 5 7	*1 2 4 5 7	*1 2 4 5 7	*1 2 4 5 7
176	*5 6 7 4 9	1.3 4 9 4 8	*1 2 3 2 5	*1 2 3 2 5	*1 2 3 2 5
177	*5 2 2 4 3	1.2 4 9 5 7	*1 2 6 6 7	*1 2 6 6 7	*1 2 6 6 7
178	*4 9 4 2 0	1.3 4 9 4 8	*2 7 2 6 2	*2 7 2 6 2	*2 7 2 6 2
179	*4 4 8 0 4	1.2 4 9 5 7	*3 3 2 3	*0 7 7 7 6	*0 7 7 7 6
180	*4 2 3 4 7	1.3 4 9 4 8	*3 0 0 1 5	*0 5 4 1 2	*0 5 4 1 2
181	*3 7 9 3 8	1.2 4 9 5 7	*3 4 0 3 5	*0 6 0 6 6	*0 6 0 6 6
182	*3 2 8 5 3	1.3 4 9 4 8	*3 2 1 6 4	*0 6 7 0 0	*0 6 7 0 0
183	*3 2 0 3 4	1.2 4 9 5 7	*3 2 5 2 2	*0 7 0 6 4	*0 7 0 6 4
184	*3 0 3 2 1	1.3 4 9 4 8	*3 2 7 1 0	*0 5 3 7 3	*0 5 3 7 3
185	*2 6 2 7 2	1.2 4 9 5 7	*3 0 6 9 4	*0 6 8 9 4	*0 6 8 9 4
186	*2 4 8 2 8	1.3 4 9 4 8	*3 4 6 7 8	*0 4 1 1 5	*0 4 1 1 5
187	*2 0 3 7 4	1.2 4 9 5 7	*3 7 5 0 4	*0 2 7 5 8	*0 2 7 5 8
188	*1 9 7 2 8	1.3 4 9 4 8	*3 5 5 0 0	*0 5 1 1 1	*0 5 1 1 1
189	*1 8 2 6 0	1.2 4 9 5 7	*3 0 1 6 9	*0 6 1 1 1	*0 6 1 1 1
190	*1 4 6 6 9	1.3 4 9 4 8	*3 0 7 0 2	*0 2 3 7 7	*0 2 3 7 7
191	*1 0 4 9 1	1.2 4 9 5 7	*3 0 9 5 3	*0 1 4 5 7	*0 1 4 5 7
192	*1 0 4 3 2	1.3 4 9 4 8	*3 0 4 7 5	*0 1 6 4 6	*0 1 6 4 6
193	*0 6 5 2 0	1.2 4 9 5 7	*3 0 6 2 1	*0 6 5 2 4	*0 6 5 2 4
194	*0 6 1 6 2	1.3 4 9 4 8	*3 0 7 1 7	*0 6 6 0 6	*0 6 6 0 6
195	*0 2 0 4 7	1.2 4 9 5 7	*3 0 9 5 3	*1 0 0 6 6	*1 0 0 6 6
196	*0 1 9 3 5	1.3 4 9 4 8	*3 6 5 9 5	*1 0 0 1 1 9	*1 0 0 1 1 9
197	*5 0 0 7 7	1.4 4 4 5 6	*3 6 4 7 5	*0 0 2 0 1	*0 0 2 0 1
198	*6 2 4 4 7	1.5 4 4 4 9	*3 2 5 2 0	*4 5 1 0 6	*4 5 1 0 6
199	*6 4 9 0 8	1.4 4 4 5 6	*1 0 7 2 1	*3 2 4 1 8	*3 2 4 1 8
200	*5 4 5 4 0	1.5 4 4 4 9	*1 1 6 9 1	*3 2 4 4 0	*3 2 4 4 0

SAMPLE PROBLEM INTEGRAL EQUATIONS

Y FLUX	P <small>R</small>	A	V <small>A</small>	V <small>L</small>	V <small>U</small>	V <small>UUKLE</small>	V <small>NDKML</small>
2.51	.014e1	1.04972	.01117	-.01117	-.01117	-.01117	.01117
2.52	.012e7	1.74245	.01242	-.01242	-.01242	-.01242	.01242
2.53	.37344	1.84761	.01111	-.01111	-.01111	-.01111	.01111
2.54	.20554	1.93523	.01111	-.01111	-.01111	-.01111	.01111
2.55	.32606	1.84765	.01111	-.01111	-.01111	-.01111	.01111
2.56	.17635	1.93533	.01111	-.01111	-.01111	-.01111	.01111
2.57	.3665	1.84765	.01111	-.01111	-.01111	-.01111	.01111
2.58	.16022	1.93533	.01111	-.01111	-.01111	-.01111	.01111
2.59	.24055	1.84767	.01111	-.01111	-.01111	-.01111	.01111
2.60	.16023	1.93533	.01111	-.01111	-.01111	-.01111	.01111
2.61	.25312	1.84762	.01111	-.01111	-.01111	-.01111	.01111
2.62	.13954	1.93532	.01111	-.01111	-.01111	-.01111	.01111
2.63	.21708	1.84765	.01111	-.01111	-.01111	-.01111	.01111
2.64	.11971	1.93533	.01111	-.01111	-.01111	-.01111	.01111
2.65	.16326	1.84762	.01111	-.01111	-.01111	-.01111	.01111
2.66	.10123	1.93533	.01111	-.01111	-.01111	-.01111	.01111
2.67	.15524	1.84764	.01111	-.01111	-.01111	-.01111	.01111
2.68	.06561	1.93533	.01111	-.01111	-.01111	-.01111	.01111
2.69	.12712	1.84765	.01111	-.01111	-.01111	-.01111	.01111
2.70	.07010	1.93533	.01111	-.01111	-.01111	-.01111	.01111
2.71	.10100	1.84765	.01111	-.01111	-.01111	-.01111	.01111
2.72	.05570	1.93533	.01111	-.01111	-.01111	-.01111	.01111
2.73	.07674	1.84765	.01111	-.01111	-.01111	-.01111	.01111
2.74	.04232	1.93533	.01111	-.01111	-.01111	-.01111	.01111
2.75	.05367	1.84764	.01111	-.01111	-.01111	-.01111	.01111
2.76	.02960	1.93533	.01111	-.01111	-.01111	-.01111	.01111
2.77	.03125	1.84765	.01111	-.01111	-.01111	-.01111	.01111
2.78	.01740	1.93533	.01107	-.01107	-.01107	-.01107	.01107
2.79	.06941	1.84765	.01090	-.01090	-.01090	-.01090	.01090
2.80	.00546	1.93533	.01091	-.01091	-.01091	-.01091	.01091

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P1.	A	Y	VX	VY	VZ	Abs. Y	C	SOURCE	V NORMAL
1.01	• ۳۱۴۵	• ۵۹۰۱۲	• ۶۷۰۰۷	• ۶۷۰۰۱	-	• ۷۰۹۴۶	-	• ۱۶۹۲۳	-
1.02	• ۳۱۲۵	• ۷۴۰۰۶	• ۶۷۰۰۵	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.03	• ۲۵۲۵	• ۶۷۰۰۵	• ۶۷۰۰۵	• ۶۷۰۰۴	-	• ۴۴۰۴۶	-	• ۱۶۹۲۴	-
1.04	• ۲۴۷۵	• ۷۴۰۰۵	• ۶۷۰۰۵	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.05	• ۱۸۲۱۴	• 6۴۰۰۵	• ۶۷۰۰۵	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.06	• ۱۸۰۳۴	• 7۴۰۰۵	• ۶۷۰۰۵	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.07	• ۱۱۴۴۹	• ۶۴۰۰۵	• ۶۷۰۰۵	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.08	• ۱۱۱۷۵	• 7۴۰۰۵	• ۶۷۰۰۵	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.09	• ۰۷۰۰۱	• 6۴۰۰۵	• ۶۷۰۰۵	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.10	• ۰۱۷۷۵	• 7۴۰۰۵	• ۶۷۰۰۵	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.11	• ۰۲۴۲۱	• 6۴۰۰۵	• ۶۷۰۰۵	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.12	• ۰۲۴۳۲	• 7۴۰۰۵	• ۶۷۰۰۵	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.13	• ۸۴۵۱۵	• 6۴۰۰۵	• ۶۷۰۰۵	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.14	• ۷۰۱۱۹	• 7۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.15	• ۸۲۳۴۷	• 8۴۰۰۵	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.16	• ۹۶۹۰۸	• 9۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.17	• 7۰۳۳۵	• ۵۴۰۰۵	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.18	• 7۱۱۵۱	• 6۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.19	• 6۹۶۴۶	• 5۴۰۰۵	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.20	• 6۷۷۰۴	• 9۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.21	• 6۰۶۶۱	• 5۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.22	• 5۸۹۰۰	• 9۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.23	• 5۴۰۳۴	• 6۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.24	• 5۰۵۶۳	• 9۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.25	• 4۴۰۰۱	• 8۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.26	• 4۶۷۷۴	• 9۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.27	• 3۷۲۱۴	• 5۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.28	• 3۰۱۷۴	• 9۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.29	• 3۰۴۷۱	• 8۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.30	• 2۹۶۶۱	• 9۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.31	• 2۴۲۱۹	• 6۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.32	• 2۲۵۳۵	• 9۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.33	• ۱۶۳۴۵	• 3۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.34	• ۱۷۸۰۲	• 9۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.35	• 1۱۳۶۴	• 6۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.36	• 1۲۵۰۵	• 9۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.37	• 0۱۷۰۲	• 5۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.38	• 1۰۳۴۵	• 9۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.39	• 0۰۳۷۴	• 8۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.40	• 0۰۲۳۳	• 3۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.41	• 8۴۱۵۹	• 1۰۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.42	• 8۰۸۴۶	• 1۱۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.43	• 8۰۲۴۱	• 1۰۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.44	• 7۱۱۵۰	• 1۱۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.45	• 7۳۶۶۸	• 1۰۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.46	• 7۰۷۹۳	• 1۱۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.47	• 6۵۴۷۹	• 1۰۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.48	• 6۰۲۹۴۰	• 1۱۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.49	• 5۱۶۲۲	• 1۰۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-
1.50	• 6۶۸۲۱	• 1۱۴۰۰۴	• ۶۷۰۰۴	• ۶۷۰۰۴	-	• ۴۶۰۴۶	-	• ۱۶۹۲۴	-

L	Flux	A	V	CP	V NORMAL
151	46.621	1.34760	46.6214	1.12	19377
152	47.074	1.14764	47.0745	1.12	18492
153	66.1304	1.07477	66.1305	1.10103	19316
154	34.765	1.14964	34.7656	1.07575	19280
155	34.975	1.07575	34.9756	1.07575	19282
156	33.667	1.04760	33.6675	1.06279	19214
157	26.647	1.64760	26.6475	1.06279	19214
158	27.557	1.14764	27.5575	1.06279	19214
159	24.762	1.04760	24.7625	1.06279	19214
160	21.879	1.14764	21.8795	1.06279	19214
161	17.244	1.04760	17.2445	1.06279	19214
162	16.624	1.14764	16.6245	1.06279	19214
163	12.074	1.04760	12.0745	1.06279	19214
164	11.665	1.14764	11.6655	1.06279	19214
165	0.71119	1.04760	0.711195	1.06279	19214
166	0.68354	1.14764	0.683545	1.06279	19214
167	0.62322	1.04760	0.623225	1.06279	19214
168	0.61466	1.14764	0.614665	1.06279	19214
169	0.77179	1.24767	0.771795	1.06279	19214
170	0.72951	1.34767	0.729515	1.06279	19214
171	0.73587	1.24767	0.735875	1.06279	19214
172	0.69544	1.34768	0.695445	1.06279	19214
173	0.67540	1.14765	0.675405	1.06279	19214
174	0.68229	1.34764	0.682295	1.06279	19214
175	0.60049	1.24767	0.600495	1.06279	19214
176	0.56749	1.34768	0.567495	1.06279	19214
177	0.52243	1.24767	0.522435	1.06279	19214
178	0.49420	1.34764	0.494205	1.06279	19214
179	0.44864	1.24767	0.448645	1.06279	19214
180	0.42339	1.34768	0.423395	1.06279	19214
181	0.37959	1.24767	0.379595	1.06279	19214
182	0.32853	1.34768	0.328535	1.06279	19214
183	0.32084	1.24767	0.320845	1.06279	19214
184	0.30321	1.34768	0.303215	1.06279	19214
185	0.26272	1.24767	0.262725	1.06279	19214
186	0.24823	1.34768	0.248235	1.06279	19214
187	0.20874	1.24767	0.208745	1.06279	19214
188	0.19723	1.34768	0.197235	1.06279	19214
189	0.15860	1.24767	0.158605	1.06279	19214
190	0.14989	1.34768	0.149895	1.06279	19214
191	0.11041	1.24767	0.110415	1.06279	19214
192	0.10462	1.34768	0.104625	1.06279	19214
193	0.05520	1.24767	0.055205	1.06279	19214
194	0.0162	1.34768	0.016215	1.06279	19214
195	0.02047	1.24767	0.020475	1.06279	19214
196	0.01935	1.34768	0.019355	1.06279	19214
197	0.68077	1.44761	0.680775	1.06279	19214
198	0.62447	1.54761	0.624475	1.06279	19214
199	0.64903	1.44763	0.649035	1.06279	19214
200	0.519540	1.54761	0.5195405	1.06279	19214
			0.719711	1.06279	1.0596
			0.716300	1.06279	1.0596
			0.716300	1.06279	1.0596

L FLUX	PI.	V A	V Y	V L	V P	SOURCE	V NORMAL
201	• 5.5674	1.44936	1.15739	1.05005	-1.020500	-2.22080	-0.67E-05
202	• 5.4607	1.54764	1.15166	1.05020	-1.035394	-2.30720	-0.82E-05
203	• 5.2956	1.46435	1.15170	1.05020	-1.035394	-1.30720	-0.60E-05
204	• 4.6526	1.54942	1.15179	1.05020	-1.035394	-1.30720	-0.40E-05
205	• 4.1125	1.44936	1.22467	1.05020	-1.035394	-1.30720	-0.20E-05
206	• 4.2311	1.54719	1.23549	1.05020	-1.035394	-1.30720	-0.10E-05
207	• 3.9572	1.44936	1.26070	1.05020	-1.035394	-1.30720	-0.10E-04
208	• 3.6330	1.24119	1.22777	1.05020	-1.035394	-1.30720	-0.10E-04
209	• 3.3463	1.44936	1.30024	1.05020	-1.035394	-1.30720	-0.10E-04
210	• 3.0690	1.54914	1.21542	1.05020	-1.035394	-1.30720	-0.94E-05
211	• 2.6330	1.44936	1.34326	1.05020	-1.035394	-1.30720	-0.97E-05
212	• 2.5929	1.54649	1.34326	1.05020	-1.035394	-1.30720	-0.93E-05
213	• 2.3173	1.44936	1.34300	1.05020	-1.035394	-1.30720	-0.97E-05
214	• 2.1257	1.54914	1.23679	1.05020	-1.035394	-1.30720	-0.96E-05
215	• 1.6412	1.44926	1.35123	1.05020	-1.035394	-1.30720	-0.93E-05
216	• 1.6409	1.54919	1.35123	1.05020	-1.035394	-1.30720	-0.93E-05
217	• 1.1982	1.44936	1.35067	1.05020	-1.035394	-1.30720	-0.98E-05
218	• 1.2432	1.54914	1.35063	1.05020	-1.035394	-1.30720	-0.98E-05
219	• 0.9763	1.44936	1.34943	1.05020	-1.035394	-1.30720	-0.97E-05
220	• 0.8974	1.54919	1.34943	1.05020	-1.035394	-1.30720	-0.96E-05
221	• 0.5751	1.44936	1.34929	1.05020	-1.035394	-1.30720	-0.98E-05
222	• 0.2275	1.54719	1.34935	1.05020	-1.035394	-1.30720	-0.98E-05
223	• 0.1806	1.44936	1.34933	1.05020	-1.035394	-1.30720	-0.97E-05
224	• 0.01656	1.54919	1.34539	1.05020	-1.035394	-1.30720	-0.97E-05
225	• 5.815	1.64342	0.94901	1.05020	-1.035394	-1.30720	-0.20E-05
226	• 4.7761	1.74343	0.924	1.05020	-1.035394	-1.30720	-0.22E-05
227	• 5.5217	1.64632	0.93814	1.05020	-1.035394	-1.30720	-0.70E-05
228	• 4.5637	1.74343	0.75442	1.05020	-1.035394	-1.30720	-0.62E-05
229	• 0.8844	1.64342	1.1943	1.05020	-1.035394	-1.30720	-0.83E-05
230	• 4.1745	1.74343	1.1943	1.05020	-1.035394	-1.30720	-0.80E-05
231	• 4.3426	1.64632	1.15319	1.05020	-1.035394	-1.30720	-0.93E-05
232	• 3.7159	1.74343	1.15213	1.05020	-1.035394	-1.30720	-0.94E-05
233	• 3.7818	1.64632	2.0602	1.05020	-1.035394	-1.30720	-0.95E-05
234	• 3.3230	1.74343	1.1721	1.05020	-1.035394	-1.30720	-0.93E-05
235	• 3.2445	1.64632	2.30114	1.05020	-1.035394	-1.30720	-0.99E-05
236	• 2.7762	1.74343	1.17613	1.05020	-1.035394	-1.30720	-0.97E-05
237	• 2.7436	1.64632	2.46117	1.05020	-1.035394	-1.30720	-0.95E-05
238	• 2.3478	1.74343	2.02014	1.05020	-1.035394	-1.30720	-0.93E-05
239	• 2.3202	1.64632	2.2089	1.05020	-1.035394	-1.30720	-0.92E-05
240	• 1.94854	1.74343	2.1902	1.05020	-1.035394	-1.30720	-0.90E-05
241	• 1.8993	1.64632	2.05337	1.05020	-1.035394	-1.30720	-0.88E-05
242	• 1.6258	1.74343	2.2707	1.05020	-1.035394	-1.30720	-0.86E-05
243	• 1.5056	1.64632	2.1106	1.05020	-1.035394	-1.30720	-0.84E-05
244	• 1.2918	1.74343	2.32913	1.05020	-1.035394	-1.30720	-0.82E-05
245	• 1.1470	1.64632	1.64342	1.05020	-1.035394	-1.30720	-0.80E-05
246	• 0.9815	1.74343	2.2425	1.05020	-1.035394	-1.30720	-0.78E-05
247	• 0.0021	1.64632	1.1514	1.05020	-1.035394	-1.30720	-0.76E-05
248	• 0.0524	1.74343	1.25154	1.05020	-1.035394	-1.30720	-0.74E-05
249	• 0.4713	1.64632	2.10413	1.05020	-1.035394	-1.30720	-0.72E-05
250	• 0.4035	1.74343	1.17447	1.05020	-1.035394	-1.30720	-0.70E-05

SAMPLE PROBLEMS TRIANGULAR ELEMENT

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ANSWER PRACTICE WORKSHEET

$$\begin{aligned} URP &= 3 \\ LGIT &= 0 \\ KEAD &= 0 \end{aligned}$$

UPP = 3

	EFF	ENDY	PLISTS	L	
T.	X	Y	000000	000000	•
1	X	000000	000000	000000	•
2	X	000000	000000	000000	•
3	X	000000	000000	000000	•

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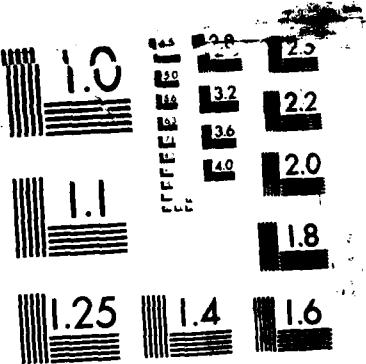
FORMULATION OF NUMERICAL METHODS USED IN THE XYZ
THREE-DIMENSIONAL POTENT. (U) TEXAS A AND M UNIV
COLLEGE STATION COLL OF ENGINEERING W J BERRY MAY 86
UNCLASSIFIED N00228-85-G-3303

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MICROCOPY RESOLUTION TEST CHART

NASA Langley Research Center

SAMPLE PROBLEMS FOR THE CELLULOSE

SAMPLE PROBLEM TRIANGULAR CLIPBOARD				UFF BOLT PINS				PALE			
FLOW	X	T	Z	X	V	Z	C/P	FLOW	X	T	Z
1	0.00000	0.00000	0.00000	-0.00000	0.00000	0.00000	0.16668	1	0.00000	0.00000	0.00000
2	0.00000	0.00000	0.00000	-0.00000	0.00000	0.00000	0.16668	2	0.00000	0.00000	0.00000
3	0.00000	0.00000	0.00000	-0.00000	0.00000	0.00000	0.16668	3	0.00000	0.00000	0.00000

SAMPLE PROBLEM TRIANGULAR ELLIPSOID

SAMPLE PROBLEMS ON TRIAXIAL ELLIPSOIDS

AMPLE POUR EN TRIAXIAL ELLIPSOIDE			URF BODY POINTS			PAGE 3		
FLD _α	X	Y	V _X	V _Y	V _Z	C.P.		
1	-0.0000	0.0000	-0.0000	-0.0000	-0.0000	-1.0000	-0.2242	-0.2242
2	-0.0000	0.0000	-0.0000	-0.0000	-0.0000	-0.7071	0.4142	0.4142
3	-0.0000	0.0000	-0.0000	-0.0000	-0.0000	-1.0000	-0.1173	-0.1173

XYZ PÜTTIIL FLJ. PKUGKAH JÉTUIK U, VETKULUH 4

ט' ינואר 1982 ע' 13000 ס' 10000

STARTING POINTS		V _A	V _Y	V _Z	C _P
PT	A	1.00000	1.00000	1.00000	0.00000
1	1.00000	1.00000	1.00000	1.00000	0.00000
2	1.00000	1.00000	1.00000	1.00000	0.00000
STEP	0				
LINE	X	Y	V _A	V _Y	C _P
1	1.00000	1.00000	1.00000	-0.2240	0.00000
2	1.00000	1.00000	1.00000	0.00000	0.00000
STEP	1				
LINE	X	Y	V _A	V _Y	C _P
1	0.95470	1.02487	0.96000	-0.4354	0.00000
2	1.02441	1.00000	0.96000	0.00000	0.00000
STEP	2				
LINE	X	Y	V _A	V _Y	C _P
1	0.91527	1.05577	0.96000	-0.5120	0.00000
2	1.05524	1.00000	0.96000	0.00000	0.00000
STEP	3				
LINE	X	Y	V _A	V _Y	C _P
1	0.88110	1.08687	0.96000	-0.5506	0.00000
2	1.08570	1.00000	0.96000	0.00000	0.00000
STEP	4				
LINE	X	Y	V _A	V _Y	C _P
1	0.85496	1.12594	0.96000	-0.5944	0.00000
2	1.12519	1.00000	0.96000	0.00000	0.00000
STEP	5				
LINE	X	Y	V _A	V _Y	C _P
1	0.63254	1.16176	0.96000	-0.4231	0.00000
2	1.17163	1.00000	0.96000	0.00000	0.00000
STEP	6				
LINE	X	Y	V _A	V _Y	C _P
1	0.61450	1.20060	0.96000	-0.4567	0.00000
2	1.12610	1.00000	0.96000	0.00000	0.00000
STEP	7				
LINE	X	Y	V _A	V _Y	C _P
1	0.79204	1.25250	0.96000	-0.4747	0.00000
2	1.08423	1.00000	0.96000	0.00000	0.00000

STEP	8	V_A	V_Y	V_L	C_P			
LINE	X 1 2	77077 1.06120	Y 1.05475 0.00000	Z 0.00000 0.00000	V_A 0.02174 0.02377	V_Y -0.02033 0.00000	V_L 0.00000 0.00000	C_P 0.69439 0.94352
STEP	9	V_A	V_Y	V_L	C_P			
LINE	X 1 2	74664 1.04141	Y 0.00000 0.00000	Z 0.00000 0.00000	V_A 0.02260 0.02400	V_Y -0.02274 0.00000	V_L 0.00000 0.00000	C_P 0.69439 0.94352
STEP	10	V_A	V_Y	V_L	C_P			
LINE	X 1 2	72070 1.02060	Y 1.04141 0.00000	Z 0.00000 0.00000	V_A 0.02702 0.02915	V_Y -0.02324 0.00000	V_L 0.00000 0.00000	C_P 0.69439 0.94352
STEP	11	V_A	V_Y	V_L	C_P			
LINE	X 1 2	69033 1.02133	Y 1.04037 0.00000	Z 0.00000 0.00000	V_A 0.02817 0.02926	V_Y -0.02360 0.00000	V_L 0.00000 0.00000	C_P 0.69439 0.94352
STEP	12	V_A	V_Y	V_L	C_P			
LINE	X 1 2	65062 1.01761	Y 1.02620 0.00000	Z 0.00000 0.00000	V_A 0.02933 0.02949	V_Y -0.02400 0.00000	V_L 0.00000 0.00000	C_P 0.69439 0.94352
STEP	13	V_A	V_Y	V_L	C_P			
LINE	X 1 2	61701 1.01586	Y 0.00000 0.00000	Z 0.00000 0.00000	V_A 0.03124 0.03113	V_Y -0.02440 0.00000	V_L 0.00000 0.00000	C_P 0.69439 0.94352
STEP	14	V_A	V_Y	V_L	C_P			
LINE	X 1 2	60676 1.01512	Y 0.00000 0.00000	Z 0.00000 0.00000	V_A 0.03206 0.03143	V_Y -0.02480 0.00000	V_L 0.00000 0.00000	C_P 0.69439 0.94352

		X	Y	Z	Vx	Vy	Vz	Cx
STEP 15								
LINE	X	1.02444	1.02444	1.02444	0.00000	0.00000	-0.00000	1.4549
1	1.01480	1.00000	1.00000	1.00000	0.00000	0.00000	0.00000	1.00000
2	1.01480	1.00000	1.00000	1.00000	0.00000	0.00000	0.00000	1.00000
STEP 16								
LINE	X	1.45208	1.74351	1.74351	0.00000	0.00000	-0.0144	0.01838
1	1.01467	1.00000	1.00000	1.00000	0.00000	0.00000	0.00000	1.00000
2	1.01467	1.00000	1.00000	1.00000	0.00000	0.00000	0.00000	1.00000
STEP 17								
LINE	X	1.37174	1.04222	1.04222	0.00000	0.00000	-0.06674	-0.16642
1	1.01461	1.00000	1.00000	1.00000	0.00000	0.00000	0.00000	1.00000
2	1.01461	1.00000	1.00000	1.00000	0.00000	0.00000	0.00000	1.00000
STEP 18								
LINE	X	1.27359	1.04247	1.04247	0.00000	0.00000	-0.0775	-0.54555
1	1.01454	1.00000	1.00000	1.00000	0.00000	0.00000	0.00000	1.00000
2	1.01454	1.00000	1.00000	1.00000	0.00000	0.00000	0.00000	1.00000
STEP 19								
LINE	X	1.05407	1.04707	1.04707	0.00000	0.00000	-0.03626	-0.6423
1	1.01456	1.00000	1.00000	1.00000	0.00000	0.00000	0.00000	1.00000
2	1.01456	1.00000	1.00000	1.00000	0.00000	0.00000	0.00000	1.00000
STEP 20								
LINE	X	0.02455	1.01458	1.01458	0.00000	0.00000	0.01133	-1.10076
1	1.01458	1.00000	1.00000	1.00000	0.00000	0.00000	0.00000	1.00000
2	1.01458	1.00000	1.00000	1.00000	0.00000	0.00000	0.00000	1.00000

XYZ POTENTIAL FLUX PREDICTION SECTION 7, VERSION 4

SAMPLE PROBLEM TRIANAL ELLIPSOID

ON ENTRY STREAMLINES - INPUT DATA

X1 =	-1.00000
Y1 =	.00000
Z1 =	.00000
NLINE =	1
JMAX =	0
LWRITE =	1
MACH NO =	.00000

STREAMLINE STARTING POINTS

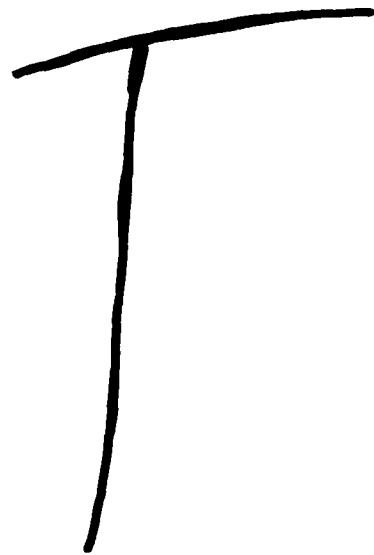
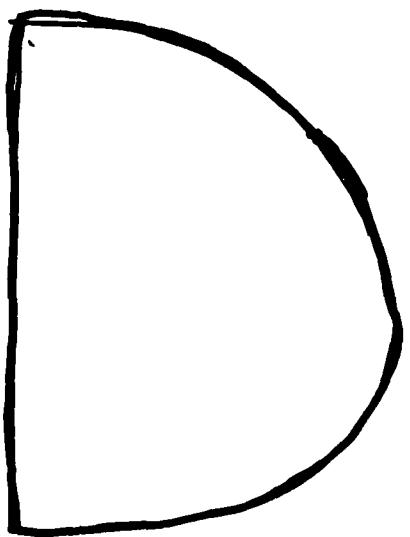
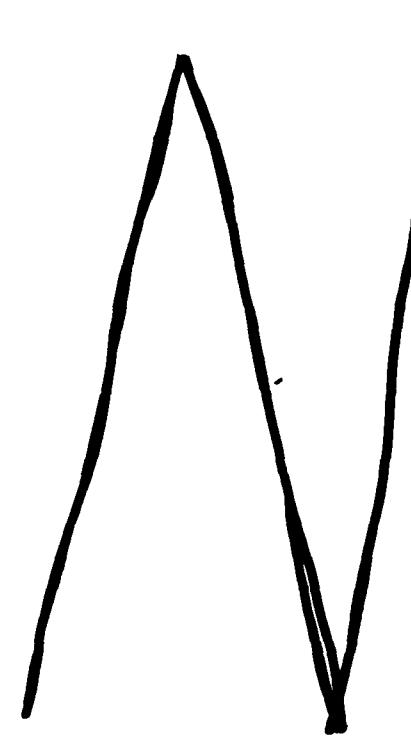
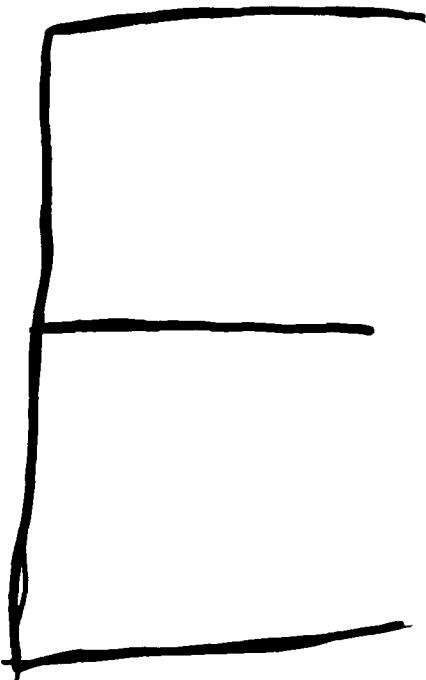
LINE	X	Y	Z
1	1.00000	.00000	.00000

SAMPLE PROBLEM TRIANAL ELLIPSOID

UNSET FLUX, X1=1.000 Y1= .000 Z1= .000

LINE NO. 1 PASSING THROUGH UNSET FLUX, 1 WITH STARTING POINT, X= 1.00000 Y= .00000 Z= .00000

1	X	Y	Z	VX	vy	vz	C1	C2	SL	V
1	.99940	.04944	-.00012	-.01221	.01221	.05148	.94649	-.10.55732	1.00000	.00000
2	.94463	.10000	.01960	-.02549	.02549	.14169	.97765	-.3.76831	1.42924	.05404
3	.94463	.10000	.01960	-.02549	.02549	.14169	.97735	-.3.76831	1.42924	.05404
1.927.20.UCLP, AA, NOTY5A,	.0442KLNS,								** LNU OF LISTING **	



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